Designs for half-diallel experiments

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Abstract

In some experiments, the experimental units are all pairs of individuals who have to undertake a given task together. If all such pairs are used exactly once each, then the set of pairs forms a triangular association scheme. If there are n individuals then there are $N = n(n-1)/2$ such pairs. The corresponding Bose–Mesner algebra has three common eigenspaces. One consists of the constant vectors (it has dimension 1); one consists of linear combinations of the indicator vectors of individuals, constrained so that the entries sum to zero (it has dimension $n-1$); the third is the orthogonal complement of these two (it has dimension $N-n$).

In classical work on design of experiments, the experimental units are grouped into b blocks of size k. Again, there are three common eigenspaces. One consists of the constant vectors; one consists of vectors which are constant on each block and whose entries sum to zero; the third is the orthogonal complement of these two.

In both cases, we assume that the variance–covariance matrix $\mathbf{C}$ of the responses to the experiment is an unknown linear combination of the matrices of projection onto these eigenspaces.

Two types of block design are particularly important. In *balanced* block designs, the variance of the estimated difference between any two treatments is the same, no matter what the eigenvalues of $\mathbf{C}$ are. In *orthogonal* block designs, the linear combination of responses which gives the best unbiased estimator of any difference between treatments does not depend on what the eigenvalues of $\mathbf{C}$ are. Such designs are often said to have *commutative orthogonal block structure*.

In this talk I concentrate on designs for half-diallel experiments. I will give some constructions for balanced designs and some for designs which have commutative orthogonal block structure.

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