EUROPT
2022 PORTUGAL

19th Workshop on Advances in Continuous Optimization
29-30 July 2022

NOVA School of Science and Technology
Universidade Nova de Lisboa, Portugal
https://sites.fct.unl.pt/europt2022/

Conference Program

EWG EUROPT CONTINUOUS OPTIMIZATION

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Stream organisers

Geovani Grapiglia, Yuriii Nesterov: Complexity of convex optimization
Mathias Staudigl: Mathematical programming and computational game theory
Markus Gabl, Tatiana Tchemisova: Conic optimization
Juan Enrique Martínez-Legaz: Continuous optimization and variational analysis
Ana Luísa Custódio, Francesco Rinaldi: Derivative-free optimization
Sonia Cafieri, Eligius Hendrix, Frédéric Messine: Global optimization
Jan Kronqvist, Sourour Elloumi: Mixed integer nonlinear optimization
Gabriele Eichfelder: Multiobjective optimization
Ioannis Baltas, Diogo Pinheiro, Gerhard-Wilhelm Weber: Optimal control and optimization in economics and social sciences
Pilar Martínez Ortigosa: Optimization on health informatics
Stefano Cipolla, Jacek Gondzio: Proximal point algorithms and related numerical methods
Akhtar Khan, Christiane Tammer: Variational inequalities and optimization under uncertainty

Overview

Friday 29 July

8:30 – 9:00	opening
9:00 – 10:00	plenary – Marc Teboulle
10:00 – 10:25	coffee break
10:25 – 11:40	parallel sessions
11:45 – 13:00	parallel sessions
13:00 – 14:30	lunch
14:30 – 16:10	parallel sessions
16:15 – 17:30	parallel sessions
17:30 – 18:00	coffee break
18:00 – 19:00	plenary – Alper Yildirim

Saturday 30 July

8:30 – 10:10	parallel sessions
10:10 – 10:40	coffee break
10:40 – 11:55	parallel sessions
12:00 – 13:00	plenary – Dolores Romero Morales
13:00 – 14:30	lunch
14:30 – 16:10	parallel sessions
16:15 – 17:30	parallel sessions
17:30 – 18:00	coffee break
18:00 – 19:00	europt fellowship lecture – Oliver Stein
19:00 – 19:15	closing

Plenaries, opening and closing: auditorium 1C
Parallel sessions: rooms 1.1, 1.2, 1.14, 1.15, 1.16, 1.17

Parallel sessions

Room	*1.1*	*1.2*	*1.14*	*1.15*	*1.16*	*1.17*
Friday 29 July						
10:25 – 11:40	MINO 1	CGT 1	CO 1	COVA 1	VIU 1	OHI 1
11:45 – 13:00	MINO 2	CGT 2	CO 2	COVA 2	VIU 2	OHI 2
14:30 – 16:10	MO 1	CGT 3	DFO 1	MINO 3	VIU 3	-
16:15 – 17:30	MO 2	CCO 1	IO 1	COVA 3	VIU 4	OC 1
Saturday 30 July						
8:30 – 10:10	MO 3	CGT 4	DFO 2	COVA 5	-	OC 2
10:40 – 11:55	MO 4	CO 3	PPA 1	COVA 4	-	OC 3
14:30 – 16:10	MO 5	CGT 5	DFO 3	GO 1	VIU 6	OC 4
16:15 – 17:30	MINO 4	CGT 6	PPA 2	GO 2	COVA 6	-

Streams

CCO – complexity of convex optimization
CGT – mathematical programming and computational game theory
CO – conic optimization
COVA – continuous optimization and variational analysis
DFO – derivative-free optimization
GO – global optimization
IO – interval optimization
MINO – mixed integer nonlinear optimization
MO – multiobjective optimization
OC – optimal control and optimization in economics and social sciences
OHI – optimization on health informatics
PPA – proximal point algorithms and related numerical methods
VIU – variational inequalities and optimization under uncertainty

Friday, 8:30 - 9:00

■ FA-01

Friday, 8:30 - 9:00 - Auditorium 1C

Opening

Stream: Plenaries
Invited session

Friday, 9:00 - 10:00

■ FB-01

Friday, 9:00 - 10:00 - Auditorium 1C

Marc Teboulle

Stream: Plenaries
Invited session
Chair: *Giancarlo Bigi*
Chair: *Sonia Cafieri*

1 - Algorithms for Structured Nonconvex Optimization
Marc Teboulle

In recent years, nonconvex optimization models have attracted a revived interest among scientists working in various disparate modern applications such as signal and image processing, and in machine learning. Indeed, in such applications, the models are often genuinely nonconvex and nonsmooth, yet they are very hard to solve. This talk will review some recent developments on the design and analysis of simple algorithms for some important classes of nonconvex problems, highlighting the pillars of the convergence theory, and the exploitation of the problem's structure and data information.

Friday, 10:25 - 11:40

■ FC-02

Friday, 10:25 - 11:40 - Room 1.1

MINO 1 - Polynomial Optimization with binary variables

Stream: Mixed-integer nonlinear optimization
Invited session
Chair: *Luca Mencarelli*

1 - Quadratization in the optimization of polynomials with binary variables
Sourour Elloumi, Yves Crama, Amélie Lambert, Elisabeth Rodriguez-Heck

We consider quadratization-based solution approaches for the optimization of polynomials of 0-1 variables. The quadratization step aims at rewriting a degree three or more pseudoboolean function into an equivalent quadratic optimization problem with binary variables. The solution step handles the obtained equivalent problem. We provide a unified presentation of quadratizations from the literature and review several methods that can be applied in the solution step. We discuss the impact of the quadratization on the efficiency of the solution step and illustrate with some computational results.

2 - Improved linear and SDP-based quadratic convex reformulations
Mathieu Verchère, Sourour Elloumi

In this talk, we focus on linear reformulations and quadratic convex reformulations for a polynomial optimization problem in binary variables. We review two linear reformulation methods, focused on maximizing the LP bound and minimizing the number of additional variables, respectively. We address some recent improvements and show that this work on linear reformulations can also enhance the performance of SDP-based quadratic convex reformulations. We present experimental results to show the practical impact of these improvements.

3 - Outer Approximation Algorithms for Binary Nonlinear Optimization Programming
Luca Mencarelli

In this talk, we consider Binary Nonlinear Optimization Problems (BNPs), i.e., optimization problems with binary variables and nonlinear objective and constraints. We consider a simple, but computationally promising convexification technique for this kind of problems and we propose multiple and single tree tailored Outer Approximation algorithms. Finally, we present computational experiments for polynomial unconstrained and constrained instances.

■ FC-03

Friday, 10:25 - 11:40 - Room 1.2

CGT 1 - Intertwining optimization, variational problems and games I

Stream: Mathematical programming and computational game theory
Invited session
Chair: *Monica Milasi*

1 - Robustness via generalized Nash games and saddlepoints
Giancarlo Bigi

Robust counterparts of optimization problems can be formulated by exploiting infinitely many constraints and semi-infinite programs can be turned into Generalized Nash games with a peculiar structure. Pairing it with a penalization scheme for GNEPs leads to a class of methods that are based on saddlepoint problems. Any algorithm for these latter problems provides the basic iterations for the penalty scheme. A projected subgradient method for nonsmooth optimization and a subgradient method for saddlepoints are adapted to our framework and the convergence of the resulting algorithms is shown.

2 - Multiple private goods to finance a public good
Marta Faias

We extend the seminal model by Bergstrom, Blume and Varian (1986) on the private provision of public goods to contributions given by multiple private commodities. Taking into account the value of the aggregate bundle to finance the public good, we present a notion of equilibrium, show its existence, and provide a strategic market game approach. We define a sequence of non-cooperative games whose equilibria converge to an equilibrium for the economy. Finally, we analyze the effects that redistributions of endowments have on the equilibrium outcome.

3 - Variational methods to study energy market equilibrium problems
Domenico Scopelliti, Monica Milasi

This talk focuses on the study of electricity market models, which evolve in T+1 stages, so that, at each stage, a continuum state of nature is possible. Each agent maximizes her utility under a budget constraint set that depends on the equilibrium price system; in turn, such a price system is determined by suitable market-clearing conditions. To capture the evolutionary aspects of the problem in response to an increasing level of information, an opportune filtered probability space is introduced and all is studied through suitable multistage stochastic quasi-variational inequality problems.

■ FC-04

Friday, 10:25 - 11:40 - Room 1.14

CO 1 - Duality, optimality conditions, and regularization in Conic Optimization

Stream: Conic optimization
Invited session
Chair: *Tatiana Tchemisova*

1 - Strong duality for standard convex programs
Qinghong Zhang, Ken Kortanek, Guolin Yu

A primal optimization problem and its dual are in strong duality if either one has a finite optimal value, then the other one is consistent and has the same value, and the dual problem is attained. We study the strong duality for a convex optimization problem using the bifunction, image space analysis, and polynomial ring approaches. Specifically, we reproduce strong duality results developed in 1970s via the bifunction approach. We use image space analysis and polynomial ring approaches to formulate new strong duals for the primal problem. Finally, we give an example to illustrate our duals.

2 - Strong dual formulations for linear copositive optimization
Tatiana Tchemisova, Olga Kostyukova

In convex and conic optimization, optimality conditions and duality are usually formulated under certain regularity conditions, constraint qualifications that ensure the fulfilment of the Karush-Kuhn-Tucker-type optimality conditions and the strong duality property, consisting in the fact that the optimal values of the primal problem and the corresponding dual Lagrange are equal and the dual problem reaches its maximum. We present new dual formulations for linear copositive problem, compare their properties and show that the obtained pairs of dual problems satisfy strong duality relations.

■ FC-05

Friday, 10:25 - 11:40 - Room 1.15

COVA 1 - Continuous optimization and variational analysis I

Stream: Continuous optimization and variational analysis
Invited session
Chair: *Juan Matías Sepulcre*

1 - Relaxed Lagrangian duality in convex infinite optimization
Miguel Goberna

We associate with each convex optimization problem P, posed on some locally convex space, and a given non-empty family H of finite subsets of its index set, a suitable Lagrangian-Haar dual problem. We present results on the equivalence of P to some subproblem obtained by replacing its whole index set by some element of H, zero duality theorems, strong and reverse strong theorems, and optimality conditions for this new type of duality.

2 - Characterizations of the set of strong Slater points of a linear semi-infinite system
Margarita Rodríguez Álvarez, José Vicente-Pérez

In this work, we study the set of strong Slater points, whose non-emptiness guarantees the fullfilment of the strong Slater condition. Given a linear inequality system, we establish some basic properties of the set of strong Slater points. Then, we derive dual characterizations for this set in terms of the data of the system, following similar characterizacions provided also for the set of Slater points and the solution set of the given system, which are based on the polarity operators for evenly convex and closed convex sets.

3 - Duality between cost functions and multi-output production mappings.
Abelardo Jordán, Juan Enrique Martínez-Legaz

In a microeconomic context of n inputs and m outputs, we present the production correspondence that maps every input vector into the set of output vectors, whose values are normal sets. From this, the cost function is derived and their properties are established. As a fundamental result, a duality scheme is formulated that involves the quasi-concave production correspondences and the cost functions, mainly by reconstructing the production correspondence from the cost function, generalizing results from single-product approach. Additionally, results about radiantness are obtained.

■ FC-06

Friday, 10:25 - 11:40 - Room 1.16

VIU 1 - Some aspects of vector optimization

Stream: Variational inequalities and optimization under uncertainty
Invited session
Chair: *Christian Günther*

1 - Active Network and Price of Anarchy in Multi-Commodity Routing Games with Variable Demands
Valerio Dose

We consider equilibria in nonatomic routing games as functions of the traffic demand in a multi-commodity network. We then study the Price of Anarchy not only as a function of every possible vector of demands across all the commodities, but also as a function of a single real parameter where the vector of demands is given by a fixed demand function of the parameter. We prove several results concerning analogies with the single-commodity setting, and the behavior of the Price of Anarchy around points where it is not differentiable. We consider both affine and more general cost functions.

2 - Explaining Trade-offs in Interactive Multiobjective Optimization
Bekir Afsar, Kaisa Miettinen

Interactive methods support decision makers (DMs) in finding the most preferred solution based on preferences provided iteratively by the DM. However, existing methods do not explain how preferences are linked to generated solutions and why they are shown to the DM. The DM should be aware of these to be confident in one's decisions. We are inspired by LIME, a well-known machine learning explainer, and use KKT multipliers to explain local trade-offs among conflicting objectives without increasing computational load. We argue that explainability has vast potential in multiobjective optimization.

3 - A Penalty Strategy Embedded in the Tchebycheff Scalarization Method for Multi-objective Optimization Problems
Ana Maria A.C. Rocha, M. Fernanda P. Costa, Edite M.G.P. Fernandes

A multi-objective optimization method based on the weighted Tchebycheff scalarization function has been used to provide reasonable solutions to a polymer screw extruder optimal design, in a way that good trade-offs between conflicting objectives are identified. In this work, a penalty term is embedded in the Tchebycheff scalarization method aiming to guide the solutions towards the Pareto front. The goal of the penalty parameter is to balance convergence and diversity. The results show that the solutions obtained by the penalty-based algorithm cover reasonably well the Pareto front.

■ FC-07

Friday, 10:25 - 11:40 - Room 1.17

OHI 1 - Optimization on health informatics I

Stream: Optimization on health informatics
Invited session
Chair: *Pilar M. Ortigosa*

1 - A new hybrid optimization algorithm to combine physical and biological criteria to compute IMRT planning
Juan José Moreno Riado, Savíns Puertas Martín, Juana Lopez Redondo, Leocadio G. Casado, Pilar M. Ortigosa, Ester M Garzon

Intensity Modulated Radiotherapy (IMRT) is an effective cancer treatment to kill tumorous cells by radiation while preserving surrounding healthy organs. Medical prescriptions must consider biological and physical criteria. Models based on the Equivalent Uniform Dose (EUD) biological criteria provide RT plans with excellent PTV coverage. However, achieving physical constraints requires additional EUD parameter tuning (EUD-PT). We propose a hybrid optimization scheme for an automatic EUD-PT, allowing a flexible combination of physical criteria and personalized prescriptions with the EUD model.

2 - A practical study of black-box and surrogate optimizers for tuning spiking neural models of striatum plasticity
N.c. Cruz, A. González-Redondo, Juana Lopez Redondo, E.m. Ortigosa, J.a. Garrido, Pilar M. Ortigosa

A realistic model of the striatum learns to select actions by trial and error. However, model tuning is challenging due to the expert knowledge required and the computational cost of assessing configurations (e.g., more than 60 minutes per run). In this work, we try to replace the tedious and potentially biased expert-based tuning with different optimizers that need few function evaluations. The options are the SurrogateOpt solver of Matlab and the RBFOpt library, both based on radial basis function approximation, and DIRECT-GL, an enhanced version of the widespread black-box method DIRECT.

3 - On the use of teaching-learning based optimization to train neural networks
Marcos Lupion Lorente, Nicolás Calvo Cruz, Ben Paechter, Pilar M. Ortigosa

Backpropagation (BP) is the most used technique to train neural networks. However, it can converge to local optima due to its mathematical conception. Meta-heuristic optimizers overcome this issue and achieve good results for small networks. This work uses and extends TLBO, one of such methods, to train a great network (LeNet-CNN). It is modified to start from a hybrid population combining pure random solutions and partially trained ones with BP. TLBO has also been updated to use BP to improve part of the population during the search, which results in a memetic optimization approach.

Friday, 11:45 - 13:00

■ FD-02

Friday, 11:45 - 13:00 - Room 1.1

MINO 2 - Mixed-integer nonlinear programming applications

Stream: Mixed-integer nonlinear optimization
Invited session
Chair: *Jan Rolfes*

1 - A Gauss-Newton-based Decomposition Algorithm for Nonlinear Mixed-Integer Optimal Control Problems
Clemens Zeile

For the fast approximate solution of Mixed-Integer Non-Linear Programs (MINLPs) arising in the context of Mixed-Integer Optimal Control Problems a decomposition algorithm exists that solves a sequence of three comparatively less hard subproblems to determine an approximate MINLP solution. In this work, we propose a problem formulation for the second algorithm stage that is a convex approximation of the original MINLP and relies on the Gauss-Newton approximation. We analyze the algorithm in terms of approximation properties and present a numerical case study of a renewable energy system.

2 - Simultaneous Design and Optimization of Complex Processes Constrained by Partial Differential Equations using Mixed Integer Formulations
Hasan Sildir, Emrullah Erturk, Sahin Sarrafi

Optimization problems constrained by partial differential equations (PDEs) are challenging but delivers significant impact in many engineering fields. A mixed-integer formulation is proposed for the simultaneous calculation of topology and process related continuous variables, unlike sequential design studies. The binary variables in the formulation limit some variables through linking constraints, forming the topology. The problem formulation is flexible and promising since it is flexible to further simplifications and reformulations in the large-scale implementations at various disciplines.

3 - Distributionally robust optimization with envelope and moment information
Jan Rolfes

Driven by an application from chromatography, we model an optimization problem with linear objective subject to robust constraints that depend on an uncertain particle size distribution (psd). Without further knowledge about this psd the problem would be too conservative. Thus, we combine a generalized form of the classical moment constraints by Shapiro with confidence set constraints on the psd. The latter are informed by an envelope around the psd. Thereby, we decrease the conservatism in our model and optimize the fractionation process together with our colleagues in process engineering.

■ FD-03

Friday, 11:45 - 13:00 - Room 1.2

CGT 2 - Intertwining optimization, variational problems and games-II

Stream: Mathematical programming and computational game theory
Invited session
Chair: *Lorenzo Lampariello*

1 - Quasi-variational problems with non-self map on Banach spaces: existence and application
Monica Milasi, Domenico Scopelliti

We study the generalized quasi-variational inequality problems with a non-self constraint map. We consider the concept of the projected solution and we prove the existence in real Banach spaces. Following the stochastic variational approach introduced by Rockafellar and Wets, we introduce the concept of the projected solution in the stochastic setting, and we prove the existence of such a solution. We apply these theoretical results in studying an electricity market with renewable power sources.

2 - Distributed methods for monotone inclusions
Yura Malitsky

In this talk, I would like to present some new methods for solving monotone inclusions with n operators. A notable interest is that they do not belong to the standard product space reformulation of existing algorithms. We will consider different methods, depending on the operators assumptions. These methods are distributed by design and can also be used for asynchronous computation. The talk is based on joint works with Matthew Tam, Francisco J. Aragón-Artacho, and David Torregrosa-Belén.

3 - Addressing Nested Variational Inequalities
Lorenzo Lampariello

We consider nested variational inequalities consisting in a (upper-level) variational inequality whose feasible region is given by the solution set of another (lower-level) variational inequality. To treat these problems numerically, we rely on the renowned Tikhonov paradigm using either averaging or prox-like techniques: convergence and complexity results are obtained under rather weak assumptions.

■ FD-04

Friday, 11:45 - 13:00 - Room 1.14

CO 2- Convex and nonconvex Optimization and applications

Stream: Conic optimization
Invited session
Chair: *Tatiana Tchemisova*

1 - Approximate solutions of convex semi-infinite optimization problems in finitely many iterations
Miltiadis Poursandis, Jochen Schmid

We develop two adaptive discretization algorithms for convex semi-infinite optimization, which terminate after finitely many iterations at approximate solutions of arbitrary precision. They terminate at a feasible point of the considered optimization problem. Compared to existing finitely feasible algorithms for general SIPs, our algorithms work with considerably smaller discretizations and all occurring finite optimization must be solved only approximately. We apply our algorithms to solve various shape-constrained regression problems in engineering processes. See: arxiv.org/abs/2105.08417.

2 - To the Solution of Constrained Experimental Design Problems - Extending an Adaptive Discretization Approach
Philipp Seufert, Jan Schwientek, Tobias Seidel, Michael Bortz, Karl-Heinz Küfer

While the literature on optimal experimental design (OED) is primarily concerned with unconstrained problems, it is also of interest to restrict individual experiments, certain design quantities, or even the entire experimental plan. For the numerical solution of such constrained infinite-dimensional nonlinear OED problems, we propose an adaptive discretization scheme w.r.t. the Lagrangian. We adapt constraint qualifications from nonlinear programming and prove convergence of the algorithm to a global optimal design. Finally, we apply the method in the context of multi-criteria OED.

■ FD-06

Friday, 11:45 - 13:00 - Room 1.16

VIU 2 - Vector optimization with uncertainties

Stream: Variational inequalities and optimization under uncertainty
Invited session
Chair: *Akhtar Khan*

1 - Computing robust efficient solutions of discrete vector optimization problems with uncertainties
Christian Günther, Nicolae Popovici, Elisabeth Anna Sophia Köbis

Different concepts of optimality are currently known in vector optimization with uncertainties (set optimization), and various types of binary relations (e.g., l-type and u-type preorder relations) can be used in order to define them. The principal aim of this talk is to present numerical procedures for computing robust efficient solutions of discrete vector optimization problems with uncertainties. These procedures are mainly based on certain set-valued counterparts of some methods originally conceived for vector optimization problems without uncertainties.

2 - Optimality conditions in optimization under uncertainty
Christiane Tammer

Most optimization problems involve uncertain data due to measurement errors, unknown future developments and modeling approximations. Stochastic optimization assumes that the uncertain parameter is probabilistic. An other approach is called robust optimization which expects the uncertain parameter to belong to a set that is known prior. In this talk, we consider scalar optimization problems under uncertainty with infinite scenario sets. We apply methods from vector optimization, set optimization and scalarization techniques to derive necessary optimality conditions.

3 - Set-valued vector optimizations in ordered Fell topological hyperspaces of partially ordered topological vector spaces
Jinlu Li

In this paper, we study the following two subjects: the continuity (topological order-embedding properties) of the canonical mappings from the partially ordered topological space X to the two ordered Fell topological hyperspaces C(X); and the application of the continuity of the mappings to prove (a) the existence of solutions of some (closed) set-valued vector optimization problems and the properties of the solutions and (b) the solvability of some (closed) set-valued vector variational inequalities.

■ FD-07

Friday, 11:45 - 13:00 - Room 1.17

OHI 2 - Optimization on health informatics II

Stream: Optimization on health informatics
Invited session
Chair: *Juana Lopez Redondo*

1 - Drugs discovery by shape similarity using Deep Learning

Luis F. Romero, Luis Felipe Romero Caparrós, Juana Lopez Redondo, Pilar M. Ortigosa

Searching for one or several molecules in a database using its shape has great interest from a biochemical point of view, but requires a huge computational effort due to the complexity of the algorithms and the sizes of the databases in the pharmaceutical industry. This work uses Deep Learning by training neural networks with hundreds of images of each molecule, rendered by projections (using GPUs) on planes whose normals are equally distributed in the 3D space (using Fibonacci spirals). The obtained results, both in accuracy and time, exceed expectations, opening a hopeful path of research.

2 - Machine Learning for the prediction of retinopathy onset

Giulia Di Teodoro, Laura Palagi, Marianna Maranghi

Type-2 diabetes is associated with microvascular complications such as retinopathy. We aim to define a predictive ML model for the onset of the pathology. The database is the unique Italian Diabetes Association collection of over 20 years. The huge dataset is extremely unbalanced: 81,867 non-retinopathic and 8,280 retinopathic patients. We first obtain a baseline predictive model applying traditional ML classification models. We propose to develop weighted ensemble models, capable of leveraging the performance of the single models to improve KPIs and to include summary features of patients.

3 - Optimization method for maximizing the similarity in flexible molecules

Savíns Puertas Martín, Juana Lopez Redondo, Valerie J. Gillet, Horacio Pérez-Sánchez, Pilar M. Ortigosa

Virtual Screening (VS) aims to reduce the time and budget required in drug discovery. Although they have several rotatable bonds, the most current VS methods assume the ligands to be rigid. Therefore, it is compulsory to consider flexibility during the screening process to achieve good predictions. In this work, we have extended the functionality of OptiPharm, a recent piece of software, by considering the flexibility of the molecules. Additionally, the new version presents new strengths regarding its previous version and can provide valuable results based on different descriptors.

Friday, 14:30 - 16:10

■ FE-02

Friday, 14:30 - 16:10 - Room 1.1

MO 1 - Continuous multiobjective optimization

Stream: Multiobjective optimization
Invited session
Chair: *Birgit Rudloff*

1 - Treating nonsmooth regularization problems via multiobjective optimization
Bennet Gebken, Katharina Bieker, Sebastian Peitz

In many applications, optimization problems arise in which a smooth, scalar objective function is regularized with a weighted (nonsmooth) regularization term. Examples are image denoising, sparse regression and machine learning. In the context of multiobjective optimization, these problems are weighted sum scalarizations of the MOP that minimizes the smooth function and the regularization term simultaneously. In this talk, we will show how this connection enables us to analyze the structure of the solution of the regularization problem and construct path-following methods for its computation.

2 - A multiobjective model predictive control algorithm
Lisa Krügel, Gabriele Eichfelder, Lars Grüne, Jonas Schießl

Model predictive control (MPC) is a control method in which an optimal control problem is solved in each iteration. In MCP it is a natural idea that we do not consider only one, but multiple objectives. This leads to the formulation of a multiobjective optimal control problem. In this talk, we introduce a multiobjective MPC scheme which yields system theoretic properties and, due to the optimization-based nature of the method, averaged and non-averaged performance results for all objective functions. Application examples and numerical simulations will illustrate our findings.

3 - A Quasi-Newton Approach for Large Scale Multi-Objective Optimization
Pierluigi Mansueto, Matteo Lapucci

We propose a Limited Memory Quasi-Newton method for large scale unconstrained multi-objective optimization. The algorithm approximates the combinations of the objective functions Hessians through positive definite matrices, whose formula is similar to the one of the L-BFGS method for scalar optimization. By means of a Wolfe line search, our method is well defined even for non-convex optimization problems. We compare the performance of the proposed algorithm with other state-of-the-art Newton and Quasi-Newton approaches, showing its effectiveness and strengths with respect to the competitors.

4 - Multivariate Dynamic Programming
Birgit Rudloff, Zachary Feinstein, Gabriela Kovacova

Dynamic multivariate problems are often time-inconsistent. Examples include the bi-objective mean-risk problem, or Nash equilibria in a dynamic game. Observing that one can formulate these problems with a set-valued value function, we extend the Bellman's principle to such value functions. It is shown that the problems above do satisfy this Bellman's principle and are thus actually time-consistent in a set-valued sense. Practical implications and interpretations are discussed. Numerical examples are given which lead to a sequence of vector optimization problems solved backwards in time.

■ FE-03

Friday, 14:30 - 16:10 - Room 1.2

CGT 3 - Mixed integer games

Stream: Mathematical programming and computational game theory
Invited session
Chair: *Tobias Harks*

1 - Submodular maximization of concave utility functions composed with a set-union operator
Fabio Furini, Stefano Coniglio, Ivana Ljubic

We study a discrete optimization problem asking for the maximization (in expectation over a discrete set of scenarios) of a concave, strictly increasing, and differentiable function (whose coefficients are scenario-dependent) composed with a set-union operator. In the work, we propose a double-hypograph decomposition which allows for projecting out the variables associated with the items by separately exploiting the structural properties of the utility function and the set-union operator.

2 - Pure Nash Equilibria in Resource Graph Games
Jannik Matuschke, Tobias Harks, Max Klimm

We study the existence of pure Nash equilibria in resource graph games, a general class of strategic games succinctly representing the players' private costs. These games are defined relative to a finite set of resources and the strategy set of each player corresponds to a set of subsets of resources. The cost of a resource is an arbitrary function of the load vector of a certain subset of resources. As our main result, we give complete characterizations of the cost functions guaranteeing the existence of pure Nash equilibria for weighted and unweighted players, respectively.

3 - Algorithmic Solutions for Almost Core Allocations
Marc Uetz

For cooperative games with empty core, several core relaxations are known, each relaxing the core minimally to make it non-empty. We consider the linear optimization problem to maximize the total amount that can be distributed over the individual players so that no proper subset of players would prefer to deviate. This can be studied also for problems with non-empty core. We discuss some complexity theoretic results for this problem, and give a 2-approximation algorithm for minimum spanning tree games. Several open questions remain. (Joint work with Rong Zou, Boyue Lin, and Matthias Walter.)

4 - Generalized Nash Equilibrium Problems with Mixed-Integer Variables
Tobias Harks

I will talk about generalized Nash equilibrium problems (GNEPs) with non-convex strategy spaces and non-convex cost functions. This general class of games includes the important case of games with mixed-integer variables for which only a few results are known in the literature. I present new approaches to characterize equilibria and show that some of them lead to direct approaches for computing them. This is joint work with Julian Schwarz.

■ FE-04

Friday, 14:30 - 16:10 - Room 1.14

DFO 1 - New algorithmic approaches in derivative-free optimization

Stream: Derivative-free optimization
Invited session
Chair: *Francesco Rinaldi*

1 - Quadratic Regularization Methods based on Finite-Difference Gradient Approximations
Geovani Grapiglia

In this talk, derivative-free quadratic regularization methods based on finite-difference gradient approximations are presented. In these methods, the accuracy of the gradient approximations and the regularization parameter in the quadratic models are jointly adjusted using a non-monotone acceptance condition for the trial points. Deterministic worst-case complexity bounds with linear dependence on the problem dimension are obtained for the generation of approximate stationary points. Preliminary numerical results illustrate the relative efficiency of the proposed methods.

2 - A derivative-free method for convex constrained minimization
Evelin Heringer Manoel Krulikovski, *Ana Luisa Custodio*, *Marcos Raydan*

We present a new DFO approach for solving convex constrained minimization problems, assuming that the feasible set is the intersection of a finite collection of convex sets. The method is based on a combination of Directional Direct Search and Spectral Projected Gradient approaches, with Dykstra's alternating algorithm used for projection, and simplex gradients replacing the true gradient. We will present the algorithmic framework, associated convergence results, and numerical experiments that state the quality of the proposed framework.

3 - An interior point method for nonlinear constrained derivative-free optimization
Andrea Brilli, *Giampaolo Liuzzi*, *Stefano Lucidi*

We consider constrained optimization problems where both the objective and constraint functions are of the black-box type. We assume that the nonlinear inequality constraints are non-relaxable, i.e. their values and that of the objective function cannot be computed outside of the feasible region. We propose a new derivative-free optimization method which is based on the use of a merit function that handles inequality constraints by means of a log-barrier approach and equality constraints by means of a quadratic penalty approach.

■ FE-05

Friday, 14:30 - 16:10 - Room 1.15

MINO 3 - Mixed-integer nonlinear optimization

Stream: Mixed-integer nonlinear optimization
Invited session
Chair: *Jan Kronqvist*

1 - Combining discrete and continuous information for multi-criteria optimization problems
Katrin Teichert, *Tobias Seidel*

For an MCO problem with a continuous underlying structure, the utility function may sometimes still be discrete in nature. A process model may have continuously dependent product yield and quality, but delivery batch size and quality categorization dictate a discrete utility function. In such a case, a Pareto front approximation that tries to represent the space of conceivable utility functions of the DM lends itself to a discrete approach, while derivative information from the continuous structure can be used to obtain better bounds. In our talk, we discuss appropriate solution strategies.

2 - Sequential penalty methods for mixed integer programs
Marianna De Santis

We present a branch-and-bound method for mixed binary programs based on adding penalty terms to the objective function along the nodes. The resulting scheme addresses a sequence of continuous problems that share the same feasible set, while the objective function slightly changes so that the penalization of the integrality constraint violation is progressively increased. In the context of mixed integer linear complementarity problems, enhancements of the method are explored and a numerical comparison with two benchmark approaches from the literature is shown.

3 - Semidefinite Programming Approach to Security Constrained Optimal Power Flow with FACTS Devices
Bartosz Filipecki

Producing accurate and secure solutions to the Optimal Power Flow problem becomes increasingly important due to rising demand and share of renewable energy sources. We consider an Optimal Power Flow model with additional decision variables associated with line switching and FACTS devices, such as phase-shifting transformers and thyristor-controlled series capacitors. We show how a Lasserre hierarchy can be applied to this model to obtain a semidefinite programming relaxation. Finally, we provide results of numerical experiments on this relaxation.

4 - On the use of regularization, trust regions, and Hessian information in outer-approximation for convex MINLP
Jan Kronqvist, David Bernal, Zedong Peng Peng, Ignacio Grossmann

Kronqvist et al. (2020) showed that by integrating ideas from level bundle methods in outer approximation (OA) it is possible to use regularization/trust-regions in the subproblems of finding new integer assignments. The same framework also enabled Hessian information to be utilized for selecting new integer assignment. Here we discuss the framework and extensions to general regularization functions. We show that finite convergence is independent of the choice of regularization function, and we present numerical results comparing different regularization functions.

■ FE-06

Friday, 14:30 - 16:10 - Room 1.16

VIU 3 - Uncertainty quantification in inverse problems

Stream: Variational inequalities and optimization under uncertainty
Invited session
Chair: *Jinlu Li*

1 - Modelling uncertainty in thermal models for residential buildings
Miguel Sama, Jorge Arias, Akhtar Khan

A new model for thermal behaviour of a single residential building which contains a single control element given by a smart thermostat is introduced. This model depends on four parameters of interest related with the insulation, heat capacity, heater power and solar energy contribution. To deal with the natural uncertainty, the mathematical model corresponds with a random differential equation where the parameters, and hence the temperature, are itself random variables. Numerical results show that enough data can compensate the lack of physical knowledge.

2 - Inferring the basal sliding coefficient for the Stokes ice sheet model under rheological uncertainty
Olalekan Babaniyi

We consider an inverse problem for the basal sliding coefficient in a nonlinear Stokes ice sheet model from noisy surface velocity measurements. This model has other rheological parameters that are unknown and uncertain but are not of interest. We use the Bayesian approximation error (BAE) approach to avoid joint inference for the Basal sliding coefficient and the rheological parameters. We show numerical examples that demonstrate the performance of the method on an ice sheet flow model problem.

3 - Inverse variational approach for a random time-dependent economic equilibrium problem
Annamaria Barbagallo

The talk deals with the random time-dependent oligopolistic market equilibrium model. In particular the policymaker's point of view of the model is studied. The random time-dependent optimal control equilibrium conditions are expressed by means of an inverse stochastic time-dependent variational inequality which is proved to be equivalent to a stochastic time-dependent variational inequality. Some existence and well-posedness results for optimal regulatory taxes are obtained. Moreover a numerical scheme to compute solutions to stochastic time-dependent variational inequalities is presented.

4 - Stochastic Approximation Approach for the Elastography Inverse Problem
Baasansuren Jadamba

We consider an inverse problem of identifying a stiffness parameter in a stochastic linear elasticity system that models displacements in human tissues under prescribed forces. An optimization approach for the problem is discussed and we present results obtained by using a projected-gradient type approximation method for the problem.

Friday, 16:15 - 17:30

■ FF-02

Friday, 16:15 - 17:30 - Room 1.1

MO 2 - Convex multiobjective optimization

Stream: Multiobjective optimization
Invited session
Chair: *Firdevs Ulus*

1 - A Benson algorithm for unbounded convex vector optimization problems
Gabriela Kovacova, Andrea Wagner, Firdevs Ulus, Birgit Rudloff, Niklas Hey

Benson's idea of approximating efficient values in the objective space has been adapted to solving bounded convex vector optimization problems in Löhne, Rudloff and Ulus (2014). In this work, we relax the assumption of boundedness. We generalize the solution concept to be applicable for both bounded and unbounded problems and derive an algorithm capable of handling both types of problems. Our algorithm initiates with a phase that simultaneously finds recession directions as well as an initial outer approximation. We illustrate the algorithm with examples.

2 - Convergence rate of Sandwiching methods for convex multi-objective optimization
Ina Lammel, Karl-Heinz Küfer

The Sandwiching method creates an inner and outer approximation of a convex Pareto front. While the algorithm has been thoroughly studied in the 2d case, no rigorous proof of the convergence rate in higher dimensions has been presented, yet. We adapt a proof of George Kamenev for a similar method for approximating convex sets to our algorithm and obtain a proof of the convergence rate for any number of objectives. We demonstrate that the smoothness conditions that yield an optimal convergence rate are fulfilled for some applications in radiotherapy planning and chemical process engineering.

3 - Direction-free primal and dual approximation algorithms for convex vector optimization problems
Firdevs Ulus, Cagin Ararat, Simay Tekgül

Various algorithms for solving convex vector optimization problems work by fixing a direction parameter. A direction-free primal algorithm has been proposed recently. We propose a direction-free geometric dual algorithm which gives finite - and finite weak '-solutions to the dual and primal problems, respectively. ' is determined by and the structure of the ordering cone. We also modify the primal algorithm such that it gives a finite (weak) -solution to both problems. We test the performance of the algorithms for randomly generated problem instances.

■ FF-03

Friday, 16:15 - 17:30 - Room 1.2

CCO 1 - Complexity of convex optimization

Stream: Complexity of convex optimization
Invited session
Chair: *Geovani Grapiglia*

1 - Stochastic minibatch subgradient projection methods for composite optimization with functional constraints
Ion Necoara

We consider optimization problems with stochastic composite objective function subject to constraints given as level sets of convex but not necessarily differentiable functions. We derive a stochastic subgradient method with random feasibility updates, taking a stochastic proximal subgradient step aimed at minimizing the objective and then a subsequent subgradient step minimizing the feasibility violation of the observed random constraints. We analyse the convergence behavior of the algorithm when the objective function is convex or has a quadratic growth, unifying nonsmooth and smooth cases.

2 - Affine-invariant contracting-point methods for Convex Optimization
Nikita Doikov, Yurii Nesterov

In this work, we present new affine-invariant algorithms for solving convex minimization problems with bounded domain. We introduce a general framework of Contracting-Point methods, which solve at each iteration an auxiliary subproblem restricting the objective onto contraction of the domain. We show that using an appropriate affine-invariant smoothness condition, it is possible to implement iteration by one step of the Taylor approximation. For the methods of order one, it recovers Frank-Wolfe algorithm. For order two, it gives new Contracting Newton method with global complexity guarantees.

3 - Asynchronous SGD Beats Minibatch SGD Under Arbitrary Delays
Konstantin Mishchenko

The existing theory predicts that Asynchronous SGD becomes arbitrarily slower than Minibatch SGD when at least one worker experiences long delays. In this work, we show that it is not true and that regardless of the worst delay, Asynchronous SGD has rates at least matching those of Minibatch SGD. We prove this for nonconvex and convex problems, which can also be either smooth or nonsmooth. In all cases, we show that the performance of Asynchronous SGD scales with the number of workers rather than the largest delay.

■ FF-04

Friday, 16:15 - 17:30 - Room 1.14

IO 1 - Interval optimization

Stream: Interval optimization
Invited session
Chair: *Miroslav Rada*

1 - Maximization of sample variance over interval data is easy on average
Miroslav Rada, Michal Cerny, Ondřej Sokol

We deal with the following NP-hard problem motivated by robust statistics: given real intervals (each of them represents an imprecise data point), maximize the sample variance over these intervals. This amounts to maximization of rank-1 convex quadratic form over a hypercube. Despite the NP-hardness, we show that when the intervals come from a fairly general probabilistic setup, the problem is solvable in almost linear time using algorithm of Ferson et al., utilizing the rank-deficiency of the underlying quadratic form. Furthermore, we show how the result could be generalized to higher ranks.

2 - gh-differentiability for Interval-valued functions: an application to fuzzy environment
Beatriz Hernández-Jiménez, Rafaela Osuna-Gómez, Tiago M. da Costa, Yurilev Chalco-Cano

A gH-differentiability notion for interval-valued functions of several real variables is presented. It overcomes some drawbacks of the gH-differentiability definition in previous literature, getting a gH-differential that is a quasilinear interval-valued function. We prove that the gH-differentiability concept given by Markov (1979), Stefanini and Bede (2009) and Stefanini and Arana-Jiménez (2019) (extensions of Gâteaux differentiability) and the new definition are equivalents. As an application, we present the extension of the new differentiability concept to the fuzzy environment.

3 - On the Properties of Interval Transportation Problems
Elif Garajová, Miroslav Rada

Interval programming provides a mathematical model for handling interval-valued uncertainty in various optimization problems. Here, we assume that only lower and upper bounds on the data are known and the values can be independently perturbed within these bounds. In this talk, we adopt the approach to model transportation problems with interval costs, supply and demand. We investigate the main properties (such as the worst optimal value or the optimal solution set) and study the computational complexity of the connected decision problems for different forms of interval transportation problems.

■ FF-05

Friday, 16:15 - 17:30 - Room 1.15

COVA 3 - Continuous optimization and variational analysis III

Stream: Continuous optimization and variational analysis
Invited session
Chair: *Abderrahim Hantoute*

1 - A variational inequality approach to Network Games
Fabio Raciti

The aim of Network Games is to model the social and economic interactions among individuals with the help of graph theoretical concepts. After the foundational paper by Ballester et al., many scholars devoted many papers to this topic, mainly using methodologies from Game Theory and focusing on the case where the solution is interior to the constraint set. In this talk we show how the variational inequality approach can be effectively used to generalize some classic results, and provide efficient computational algorithms.

2 - Properties of the level sets of functions whose critical sets and Hess(+) complements are bounded
Cornel Pintea

We investigate the connectedness and the convexity of the level sets of the real valued functions, of two and three variables, with bounded critical sets and bounded Hess(+) complements. Recall that the Hess(+) region of the real valued function f is the set of all points in the domain of f such that the Hessian matrix of f is positive definite and its critical set of the is the vanishing set of its gradient. The technical details are based on the extreme points of the curvature function associated to f and they are pointed out by using the Lagrange multipliers technique.

3 - Tour on robust stability, duality and optimal conditions
Abderrahim Hantoute

In this talk we discuss some new optimality conditions for robust convex optimization on locally convex spaces. Our conditions make use of continuity and uniform continuity type criteria. These results are then applied to introduce new robust dual problems and provide operative conditions that ensure zero-duality gaps, stable duality and strong duality. Our approach make use of subdifferential calculus of integral functions, series functions, and infinite/robust sums of convex functions.

■ FF-06

Friday, 16:15 - 17:30 - Room 1.16

VIU 4 - Robust optimization

Stream: Variational inequalities and optimization under uncertainty
Invited session
Chair: *Annamaria Barbagallo*

1 - On proper minimality in robust vector optimization: a set optimization approach
Elena Molho, Lidia Huerga, Enrico Miglierina, Vicente Novo

We extend some notions of proper minimality to set optimization. We focus our attention on the concepts of Henig proper minimality, where stability with respect to perturbations of the ordering cone is considered, and Geoffrion proper minimality, where the boundedness of trade off-ratios between conflicting objectives is required. We compare the two classes of notions in a finite dimensional space ordered by a special class of polyhedral cones. Both sufficient and necessary optimality conditions for proper minimality are proved by using linear and nonlinear scalarizations.

2 - Scalarization and robustness in uncertain vector optimization problems
Elisa Caprari, Lorenzo Cerboni Baiardi, Elena Molho

The robust optimization approach can be used to tackle uncertain vector problems by considering worst case scenarios. In this context, notions of robust efficient solutions which are coherent with a set-valued minimization process have been introduced and we address the question whether scalarization and robustification can be commuted in a non componentwise framework.

3 - Scalarization and robustness in games with uncertain vector payoffs
Lorenzo Cerboni Baiardi, Elena Molho, Elisa Caprari

In games with vector payoffs, players evaluate their outcomes characterized by conflicting objectives.When payoffs are affected by uncertain parameters, the robust optimization approach a' la Ben Tal and Nemirowsky can be used to tackle such indeterminacy. Robust Nash equilibria extend the classical notion of equilibria in this context.We consider an axiomatic approach to scalarization that allows us to identify (pure and mixed) Nash equilibria in non cooperative games with vector payoffs.In this light, we discuss different approaches to scalarization that have been proposed in literature.

■ FF-07

Friday, 16:15 - 17:30 - Room 1.17

OC 1 - Optimal control and optimization in finance, commodity trade, insurance and pension fund systems I

Stream: Optimal control and optimization in economics and social sciences
Invited session
Chair: *Nuno Azevedo*

1 - Bilevel interior point differential dynamic programming
Andrei Pavlov, Jia-Jie Zhu

We present a novel interior-point differential dynamic programming algorithm for solving bilevel optimization problems that arise in optimal control of nonlinear discrete-time dynamics with general inequality constraints. Our primary motivation is to solve optimal control problems in the presence of external disturbances or adversaries. We demonstrate how exploiting the structure of the optimization problem given by the Markovian property, Bellman optimality principle, and interior point method leads to a computationally efficient numerical algorithm for the bilevel optimal control problem.

2 - A Dynamic Programming Approach for a Nonzero Sum Stochastic Differential Game Problem
Gerhard-Wilhelm Weber, Emel Savku

We apply dynamic programming principle to discuss an optimal investment problem by using nonzero-sum stochastic game approach in a continuous-time Markov regime-switching environment within the frame work of behavioral finance. We represent different states of an economy and, consequently, investors' floating levels of psychological reactions by a D-state Markov chain. We derive regime-switching Hamilton-Jacobi-Bellman-Isaacs equations and obtain explicit optimal portfolio strategies with Feynman-Kac representations of value functions.

3 - Two-player zero-sum stochastic differential games with Markov-switching jump-diffusion dynamics and a random horizon
Nuno Azevedo, Miguel Ferreira, Diogo Pinheiro, Susana Pinheiro

We consider a two-player zero-sum stochastic differential game with a random horizon whose state variable dynamics are given by a Markov-switching jump-diffusion. We study this game using a combination of dynamic programming and viscosity solution techniques. Under some mild assumptions, the value of the game exists and is the unique viscosity solution of a certain nonlinear partial integro-differential equation of Hamilton-Jacobi-Bellman-Isaacs type. Time allowing, we will discuss applications to Finance and Insurance.

Friday, 18:00 - 19:00

■ FG-01

Friday, 18:00 - 19:00 - Auditorium 1C

Alper Yildirim

Stream: Plenaries
Invited session
Chair: *Paula Amaral*

1 - Convex Relaxations of Nonconvex Quadratic Programs: A New Perspective via Convex Underestimators
E. Alper Yildirim

Quadratic programming is concerned with minimizing a (nonconvex) quadratic function over a polyhedron. In addition to numerous applications, quadratic programs arise as subproblems in many algorithms for general nonlinear optimization problems. It is therefore a fundamental NP-hard problem in optimization.

Saturday, 8:30 - 10:10

■ SA-02

Saturday, 8:30 - 10:10 - Room 1.1

MO 3 - Mixed-integer multiobjective optimization

Stream: Multiobjective optimization
Invited session
Chair: *Gabriele Eichfelder*

1 - Solving multi-objective mixed-integer convex optimization problems by hybrid patch decomposition
Leo Warnow, Gabriele Eichfelder

In this talk, we present a new approach to compute an enclosure of the nondominated set of multi-objective mixed-integer convex optimization problems. More precisely, we decompose the overall problem into several multi-objective continuous convex optimization problems, which we refer to as patches. We then dynamically compute and improve coverages of the nondominated sets of those patches to finally combine them to obtain an enclosure of the nondominated set of the original problem. Further, we introduce a mechanism to reduce the number of patches that need to be considered in total.

2 - Multiobjective Optimization of Decentralized Energy Supply Networks
Moritz Link, Stefan Volkwein

In view of the climate change, energy supply network planning can no longer focus only on the economic objective, but also on other criteria such as minimizing the carbon emissions. This leads to multiobjective optimization problems for decentralized energy supply networks. Moreover, taking into account static energy flow equations together with modeling practical decisions in the network yields a mixed-integer nonlinear structure of the resulting problems. In this talk, we present first results towards tackling real-world network instances.

3 - Adaptive patch approximation algorithm for bicriteria mixed-integer problems
Erik Diessel

We present an algorithm for bicriteria mixed-integer problems with convex constraints based on creating patches consisting of solutions with shared assignments for the discrete variables. By solving single-criteria MIPs, iteratively patches are computed that provide the largest possible improvement on the approximation quality. We show that the algorithm requires a number of iterations that is of the same order as the minimal number of patches necessary to achieve the given approximation quality. This is illustrated by bicriteria optimization of supply chains regarding costs and risks.

4 - Using Dual Bounds for Multiobjective Mixed Integer Quadratic Programming
Daniele Patria, Marianna De Santis, Gabriele Eichfelder, Leo Warnow

We present an algorithmic framework for solving Multi-Objective Mixed Integer Convex Quadratic Programs. The framework is based on two phases: a branch-and-bound method is applied to detect promising integer fixings, that are needed to initialize the second phase, where the Pareto front is detected using the solver HyPaD. The branch-and-bound works by fixing subsets of variables to integer values. Lower bounds are computed by addressing properly defined dual problems. A smart preprocessing allows to enumerate nodes in the branch-and-bound tree very quickly. Numerical experiments are shown.

■ SA-03

Saturday, 8:30 - 10:10 - Room 1.2

CGT 4 - Advances in conic optimization

Stream: Mathematical programming and computational game theory
Invited session
Chair: *Alice Calamita*

1 - A Sparse Interior Point Method for Linear Programs arising in Optimal Transport
Filippo Zanetti, Jacek Gondzio

Discrete Optimal Transport (OT) problems give rise to very large linear programs with a particular structure of the constraint matrix. In this talk we present an Interior Point Method (IPM) specialized for OT. Its main features are: we force all intermediate iterates to be as sparse as possible; we employ a matrix-free technique to avoid forming the huge matrices involved; we solve the Newton system mixing direct and iterative solvers. We compare our method with state-of-the-art network solvers and show that the IPM can compete with them in terms of computational time and memory usage.

2 - Computational comparison of various formulations of MIQP problems
Alice Calamita, Pasquale Avella, Laura Palagi

We discuss different formulations that can be used to address a family of quadratic mixed-integer optimization problems with logically constrained decision variables. In particular, we consider problems that exhibit logical relationships between continuous and discrete variables and present quadratic convex objectives. The formulations are compared from a computational point of view on problems arising in network applications. Differences in terms of size, computing times, and other optimization performance are highlighted.

■ SA-04

Saturday, 8:30 - 10:10 - Room 1.14

DFO 2 - Derivative-free optimization for challenging problems

Stream: Derivative-free optimization
Invited session
Chair: *Ana Luisa Custodio*

1 - A derivative-free method for stochastic structured optimization problems
Andrea Cristofari, Francesco Rinaldi

In this talk, we propose a derivative-free method to minimize the expected value of a function over the convex hull of a finite number of atoms. The proposed algorithm uses random estimates of the objective function and tries to solve, at each iteration, a reduced problem obtained by considering only a subset of atoms. Under standard assumptions on the accuracy and the variance of the random estimates, global convergence to stationary points is established. Finally, we provide some numerical results.

2 - Comparison of Randomized Direct-Search Approaches: Application to Beam Angle Optimization in Intensity-Modulated Proton Therapy
Humberto Rocha, Joana Matos Dias

Direct-search approaches have been used to successfully address challenging real-world optimization problems. Randomization of poll directions is one of the strategies for computational time efficiency. Opportunistic polling on a positive spanning set is an effective deterministic strategy to reduce the number of function evaluations. Recently, probabilistic strategies reported excellent numerical results by testing a reduced number of random poll directions at each iteration. In this study we compare different randomized direct-search approaches applied to beam angle optimization in IMPT.

3 - Effective matrix adaptation strategy for noisy derivative-free optimization
Morteza Kimiaei, Arnold Neumaier

In this paper, we construct and implement two effective versions of the matrix adaptation evolution strategy (MAES) for noisy derivative-free optimization problems. The main ingredients of our method are a new stochastic non-monotone line search condition and a new subspace technique. A comparison with state-of-the-art solvers show that our method is highly competitive in the presence of strong noise, having the lowest relative cost of function evaluations and the highest number of solved problems.

4 - Universal adversarial perturbation via generalised matrix norms
Dayana Savostianova, Francesco Tudisco

In recent years DNNs became wide-spread in computer vision problems. Although quite effective for those tasks they are vulnerable to adversarial attacks. In this talk, we examine a simple and yet effective approach based on a nonlinear extension of the classical matrix power method for creating Universal Adversarial perturbations. In particular we discuss possible extensions of this method that incorporate the knowledge of intrinsic and structural non-homogeneity of the real data resulting in less perceptible noise.

■ SA-05

Saturday, 8:30 - 10:10 - Room 1.15

COVA 5 - Continuous optimization and variational analysis V

Stream: Continuous optimization and variational analysis
Invited session
Chair: *Francisco Javier Aragón Artacho*

1 - Exponential polynomials via convex hulls
Juan Matías Sepulcre

In this talk we present an overview on the global distribution of the zeros of exponential polynomials with complex coefficients and frequencies, in which the convex hulls of the complex conjugates of the frequencies play a key role. As a continuation of these investigations, we analyze the density properties of the projections of the zeros of certain classes of exponential sums, and other related functions such as almost periodic functions, which is a topic with applications in many fields. Also, we present a thorough extension of Bohr's equivalence theorem for these classes of functions.

2 - On farthest Bregman Voronoi cells
Juan Enrique Martínez-Legaz, Maryam Tamadoni Jahromi, Eskandar Naraghirad

Let g be a strictly convex function on an evenly convex set X with nonempty interior. Assuming that g is differentiable on the interior of X, we consider the Bregman distance associated with g. Given a set T, whose elements are called sites, and a particular site s, the farthest g-Bregman Voronoi cell of s consists of all points that are farther from s than from any other site, with respect to the Bregman distance associated with g. We study farthest g-Bregman Voronoi cells; in particular, we characterize those sets that can be written as the farthest g-Bregman Voronoi cell of s for some T.

3 - Set-valued evenly convex functions: characterizations and c-conjugacy
Maria Dolores Fajardo

We deal with set-valued functions with values in the power set of a separated locally convex space where a nontrivial pointed convex cone induces a partial order relation. A set-valued function is evenly convex if its epigraph is an evenly convex set, i.e., it is the intersection of an arbitrary family of open half-spaces. We characterize evenly convex set-valued functions as the pointwise supremum of its set-valued e-affine minorants. Moreover, a suitable conjugation pattern will be developed for these functions, as well as the counterpart of the biconjugation Fenchel-Moreau theorem.

4 - Duality assertions in vector optimization w.r.t. relatively solid convex cones in real linear spaces
Bahareh Khazayel, Christian Günther, Christiane Tammer

We derive duality assertions for vector optimization problems in real linear spaces based on a scalarization using the concept of relative solidness for convex cones. We focus on an abstract vector optimization problem with generalized inequality constraints and investigate Lagrangian type duality assertions for (weak, proper) minimality notions. Our interest is neither to impose a pointedness assumption nor a solidness assumption for the convex cones involved in the solution concept of the vector problem. We are able to extend the well-known Lagrangian vector duality approach by Jahn (1983).

■ SA-07

Saturday, 8:30 - 10:10 - Room 1.17

OC 2 - Optimal control and optimization in finance, commodity trade, insurance and pension fund systems II

Stream: Optimal control and optimization in economics and social sciences
Invited session
Chair: *Emel Savku*
Chair: *Miguel F. Anjos*

1 - An Inexpensive Machine Learning Approach for Robust Forecasting, or How To Fix the Forecasting Models that the Pandemic Broke
Miguel F. Anjos

Reliable forecasts are a crucial business need. The Covid-19 pandemic's effect on consumer behaviour caused many predictive models to completely fail. We present a machine learning method developed with ExPretio Technologies to tackle this problem for rail passenger demand forecasting. The key insight is that existing models can be reused as inputs for machine learning to reassess their predictions in the context of new data. This method not only fixed the models broken by the pandemic but also gives an improved performance compared to previously used individual models on pre-pandemic data.

2 - Market Integration of Behind-the-meter Residential Energy Storage
Bárbara Rodrigues, Miguel F. Anjos, Valérie Provost

This work proposes an innovative business model to harness the potential of aggregating behind-the-meter residential storage in which the aggregator compensates prosumers for using their energy storage system. A bilevel model is developed to evaluate the potential of this business model and determine the optimal compensation scheme for the participants. A realistic Texas case study confirms the year-round profitability of the model, and that the main driver for a successful implementation is a suitable setting of the compensation paid to participants for using their energy storage system.

3 - Vehicle Routing Heuristics based on Reinforcement Learning
Corrado Coppola, Laura Palagi, Simone Foà, Giorgio Grani

We address one of the hardest routing problem, the VRP, from an optimal control perspective. We cast the problem into a Markov Decision Process environment, and we integrate the system dynamic with standard improving heuristics. We use Reinforcement Learning algorithms, in particular Proximal Policy Gradient, to train a neural architecture which approximates the stochastic optimal policy. The tests we are carrying out are showing early promising results with respect to state-of-the-art solvers.

4 - Adjoint state method for the trajectory optimization of a reentry vehicle
Francesco Marchetti, Edmondo Minisci

The presented work investigates the use of the adjoint state method for the trajectory optimization of a reentry vehicle. The trajectory is optimized using a direct optimal control method with single shooting transcription, where the gradients are evaluated using the adjoint method. This approach leads to a faster optimization process with results comparable to those obtained using the finite differences. The comparison between the adjoint method and the use of the finite differences will be detailed, both in terms of gradient computations and optimisation efficiency.

Saturday, 10:40 - 11:55

■ SB-02

Saturday, 10:40 - 11:55 - Room 1.1

MO 4 - Global multiobjective optimization

Stream: Multiobjective optimization
Invited session
Chair: *Gabriele Eichfelder*

1 - Conditions for the existence of weakly efficient solutions to vector optimization problems
César Gutiérrez, Ruben Lopez

The talk addresses the existence of weakly efficient solutions in vector optimization. By connecting the notion of colevel of a vector mapping with a formulation of the so-called Gerstewitz scalarization, new existence results are obtained in different settings. In particular, the noncoercive and nonconvex case is dealt with by considering a suitable regularization of the nominal problem.

2 - Numerical certification of Pareto optimality for biobjective nonlinear problems
Frédéric Messine, Charles Audet, Jordan Ninin

The solution to a biobjective optimization problem is composed of a collection of trade-off solution called the Pareto set. Based on a computer assisted proof methodology, the present work studies the question of certifying numerically that a conjectured set is close to the Pareto set. Two situations are considered. First, we analyze the case where the conjectured set is directly provided: one objective is explicitly given as a function of the other. Second, we analyze the situation where the conjectured set is parameterized: both objectives are explicitly given as functions of a parameter.

3 - Conic reformulations for quadratic multiobjective optimization
Gabriele Eichfelder, Patrick Groetzner

In a single-objective setting, nonconvex quadratic problems can equivalently be reformulated as convex problems over the cone of completely positive matrices. Considering multiobjective non-convex quadratic problems, naturally the question arises whether the advantage of convex reformulations extends to the multicriteria framework. Within this talk we show that this approach is no longer useful for two or more objective functions. The reason it that it only allows to find the supported nondominated points, which can, however, already be found by using a weighted sum scalarization.

■ SB-03

Saturday, 10:40 - 11:55 - Room 1.2

CO 3- Advances in copositive and quadratic optimization

Stream: Conic optimization
Invited session
Chair: *Markus Gabl*

1 - Scalable copositive methods for two-stage-stochastic StQPs
Immanuel Bomze, Markus Gabl, Francesca Maggioni, Georg Pflug

We study two-stage stochastic optimization of (nonconvex) Standard Quadratic Optimization Problems (StQPs) and treat it by discretization, leading to nonconvex quadratic problems over polytopes. By a Pairwise Frank-Wolfe method, we generate high-quality feasible solutions quickly (and more generally, guaranteed in finite time). Rigorous scalable lower bounds are obtained by refined copositivity techniques.

2 - Conic formulation of QPCCs applied to truly sparse QPs
Bo Peng, Immanuel Bomze

We study (nonconvex) quadratic optimization problems with complementarity constraints, showing an exact completely positive reformulation of it under mild conditions involving only the constraints. Moreover, we also give the conditions for strong conic duality between the obtained completely positive problem and its dual. An application on pursuing interpretable sparse solutions of quadratic optimization problems is shown to satisfy our settings, and therefore we link quadratic problems with an exact sparsity term to copositive optimization.

3 - Sparse Conic Reformulation of Structured QCQPs based on Copositive Optimization
Markus Gabl

Motivated by recent developments in search of more practical conic reformulations of QCQPs, we investigate sparse conic reformulations of QCQPs with arrowhead sparsity patterns. We present a new type of convex reformulation of QCQPs that lifts the variables into a comparatively lower dimensional space. It rests on a generalization of the set-completely positive matrix cone, that can be approximated via inner and outer approximations in order to obtain bounds, which potentially close the optimality gap.

■ SB-04

Saturday, 10:40 - 11:55 - Room 1.14

PPA 1 - Splitting and regularization

Stream: Proximal point algorithm and related numerical methods
Invited session
Chair: *Stefano Cipolla*

1 - Convergence of Douglas-Rachford splitting for nonmonotone inclusions
Brecht Evens, Pieter Pas, Puya Latafat, Panagiotis Patrinos

We study the classical Douglas-Rachford splitting (DRS) method in the absence of the monotonicity assumption. Introducing the concept of (σ,ρ)-semimonotonicity, we investigate DRS for the sum of two general semimonotone operators and show convergence of DRS with appropriately chosen parameters, even when the sum of the two operators (or of their inverses) is nonmonotone. Our analysis relies on establishing a connection with the proximal point algorithm, allowing us to show global convergence of DRS under an oblique weak Minty condition on the underlying primal-dual operator.

2 - Iterative regularization for low complexity regularizers
Silvia Villa, Mathurin Massias, Cesare Molinari, Lorenzo Rosasco

Iterative regularization exploits the implicit bias of an optimization algorithm to regularize ill-posed problems. Constructing algorithms with such built-in regularization mechanisms is a classic challenge in inverse problems but also in modern machine learning, where it provides both a new perspective on algorithms analysis, and significant speed-ups compared to explicit regularization. In this talk, I will discuss an iterative regularization procedure able to handle biases described by non smooth and non strongly convex functionals, prominent in low-complexity regularization.

3 - Relaxed Peaceman-Rachford splitting method to find the zero of the sum of two maximal strongly monotone operators: Convergence and its rate
Chee Khian Sim

We consider the problem to find zeros of the sum of two maximal monotone operators. Solving this problem has wide applicability, as the problem appears in many areas, such as in machine learning for example. The relaxed Peaceman-Rachford splitting method is one of the known methods to solve the problem. In this presentation, we give theoretical results on the convergence and the convergence rate of the method to find the zero of the sum of two maximal strongly monotone operators for different values of the relaxation parameter.

■ SB-05

Saturday, 10:40 - 11:55 - Room 1.15

COVA 4 - Continuous optimization and variational analysis IV

Stream: Continuous optimization and variational analysis
Invited session
Chair: *Miguel Goberna*

1 - A primal-dual splitting algorithm for composite monotone inclusions with minimal lifting
Francisco Javier Aragón Artacho, Radu Ioan Bot, David Torregrosa-Belén

We present a resolvent splitting algorithm for solving composite monotone inclusion problems, whose objective is finding a zero in the sum of maximally monotone operators composed with linear operators. Our main contribution is establishing the first primal-dual splitting algorithm for composite monotone inclusions with minimal lifting. Specifically, the proposed scheme reduces the dimension of the product space where the underlying fixed point operator is defined, in comparison to other algorithms, without requiring additional evaluations of the resolvent operators.

2 - Forward-Backward Methods with Reduced Dimensions
David Torregrosa-Belén, Francisco Javier Aragón Artacho, Yura Malitsky, Matthew Tam

In this talk, we present two new forward-backward-type algorithms for finding a zero in the sum of finitely many monotone operators, which are not based on reduction to a two operator inclusion in the product space. These methods reduce the dimension of the fixed point operators, in comparison to other forward-backward schemes without requiring additional evaluations of the operators.

3 - Splitting and projection methods for control-constrained linear-quadratic optimal control problems
Bethany Caldwell, Regina Burachik, Yalçın Kaya

We use projection-type algorithms to solve control-constrained linear-quadratic optimal control problems. Instead of the traditional approach of discretizing and solving the problem via large-scale numerical optimization techniques, we split the problem in two and use projection methods. Promising numerical results with this approach for a double integrator problem have earlier been obtained by Bauschke, Burachik and Kaya. Here we extend their work to more general linear-quadratic optimal control problems and provide numerical results and comparisons using four different algorithms.

■ SB-07

Saturday, 10:40 - 11:55 - Room 1.17

OC 3 - Optimal control and optimization in finance, commodity trade, insurance and pension fund systems III

Stream: Optimal control and optimization in economics and social sciences
Invited session
Chair: *Gerhard-Wilhelm Weber*

1 - Optimization of Trading Signals in Intraday Pairs Trading Strategy
Petra Tomanová, Vladimír Holý

We present a stochastic spread approach to the intraday pairs trading strategy using ultra-high-frequency data. The spread process is modeled by the Ornstein-Uhlenbeck process with continuous time. To avoid bias in parameter estimation due to the microstructure noise of the ultra-high-frequency data, we propose three noise-robust estimators. For the optimal trading signals, we focus on the mean-variance optimization and determine the first-passage times of the process. We also address the geometric and optimization-theoretic viewpoint of the risk-bounded trading strategy.

2 - Facility Locations on a Network
Mindaugas Kepalas, Julius Žilinskas

We consider the problem of locations on a network: we want to open K facilities, with the aim to place them in such a way that a certain criterion is minimal. Usually, the facilities may be located in a continuous space or in a discrete set of possible places. In our problem, we instead require that each facility must be located on an edge of a planar network. In addition, we add a constraint that facilities cannot be too close to each other: there is a certain given radius R, which must separate any two facilities. We present the results of our research in this presentation.

Saturday, 12:00 - 13:00

■ SC-01

Saturday, 12:00 - 13:00 - Auditorium 1C

Dolores Romero Morales

Stream: Plenaries
Invited session
Chair: *Sonia Cafieri*
Chair: *Giancarlo Bigi*

1 - Transparent Machine Learning Calls For More Optimization
Dolores Romero Morales

While the complexity of data and models is continuously increasing, the use of machine learning in high-stakes decisions calls for more transparency and accountability. In this presentation we will navigate through some novel mathematical optimization models that embed explainability and fairness in the construction of Data Science models. This includes the ability to provide global, local and counterfactual explanations, as well as model cost- sensitivity and fairness requirements. We will show the versatility of our methodology when applied to more complex data types such as functional data.

Saturday, 14:30 - 16:10

■ SD-02

Saturday, 14:30 - 16:10 - Room 1.1

MO 5 - Multiobjective optimization and machine learning

Stream: Multiobjective optimization
Invited session
Chair: *Julia Niebling*
Chair: *Gabriele Eichfelder*

1 - A multiobjective view on creating counterfactual explanations for explaining uncertainty in machine learning
Julia Niebling

Counterfactual explanations are a popularly used technique in explainable and interpretable AI. For a given data sample and a trained machine learning model, they provide a new, slightly different data sample together with its new prediction. Recently, counterfactual explanations are also used to explain uncertainty. For that, an optimization problem has to be solved in order to minimize the uncertainty as well as the distance to the original data. This talk gives a multiobjective view on the described problem setting as this provides more insights to the data and its uncertainty.

2 - Deep Learning the Efficient Frontier of Convex Vector Optimization Problems
Zachary Feinstein, *Birgit Rudloff*

In this talk, we propose a neural network architecture to approximate the weakly efficient frontier of convex vector optimization problems satisfying Slater's condition. The proposed methodology provides both an inner and outer approximation of the weakly efficient frontier. The proposed machine learning methodology provides an upper bound to the error at each approximated efficient point. In numerical case studies we demonstrate that the proposed algorithm is effectively able to approximate the true weakly efficient frontier of convex vector optimization problems.

3 - Series of Hessian-Vector Products for Tractable Saddle-Free Newton Optimisation of Neural Networks
Ross Clarke, *Elre Oldewage*, *Jose Miguel Hernandez Lobato*

Saddle-Free Newton methods offer an appealingly intuitive strategy for second-order optimisation, but require direct modification of the eigenvalues of the Hessian (or an approximation thereof). We present a series-based approach to this eigenvalue transformation which avoids the need to eigendecompose the Hessian, and can be computed efficiently by Hessian-vector products alone. Our truncated series provides an interesting new method for tractable exploitation of the Hessians of large neural network models; in its untruncated form, it converges exactly to the inverted saddle-free Hessian.

4 - Training neural networks with l1 regularization via multiobjective continuation
Katharina Bieker, *Bennet Gebken*, *Sebastian Peitz*

Regularization with an l1-norm term is common practice in many areas, e.g. in neural network training. There, the motivation is to ensure robustness against noisy data and to avoid overfitting. In order to gain a better understanding of the trade-off between accuracy and sparsity, we will not use the standard weighted-sum approach. Instead, we solve the corresponding multiobjective optimization problem. We present a continuation method which is specifically tailored to the l1-norm as a second objective and discuss the advantages but also challenges of our method in neural network training.

■ SD-03

Saturday, 14:30 - 16:10 - Room 1.2

CGT 5 - Advances in operator splitting

Stream: Mathematical programming and computational game theory
Invited session
Chair: *Pontus Giselsson*

1 - Double Splitting Preconditioner A New Class of Preconditioners
Maycon De Souza, *Aurelio Oliveira*

We propose a generalization of splitting preconditioner. We propose a double preconditioner for the augmented system, including three new parameters. In this way, it allowed us a greater range of possibilities for new preconditioners, which we named Double Splitting Preconditioner. Similar to the previous one, the double splitting preconditioner works very well close to a solution to the LP problem, a nice feature, since close to the solution the system tends to be extremely ill-conditioned. We perform some tests using the Matlab software, to verify the efficiency of the proposed method.

2 - Escaping limit cycles: Global convergence for constrained nonconvex-nonconcave minimax problems
Puya Latafat, *Thomas Pethick*, *Panagiotis Patrinos*, *Olivier Fercoq*, *Volkan Cevher*

It is well-known that finding a local solution for general minimax problems is computationally intractable. This observation has motivated the study of structures sufficient for convergence of first order methods when the so-called weak Minty variational inequality (MVI) holds. This problem class captures non-trivial structures as we demonstrate with examples, for which a large family of existing algorithms provably converge to limit cycles. Our results require a less restrictive parameter range in the weak MVI compared to what is previously known, thus capturing a larger class of problems.

3 - Inertial Algorithms in optimization, variational inequalities and fixed point problems
Juan Peypouquet

We present an overview of the dynamical aspects of old and new first-order methods used in optimization and variational analysis, and how inertial features and relaxation can help improve their performance. Special attention will be paid to inertial and overrelaxed primal-dual methods, as an illustration.

4 - Necessary and sufficient conditions for existence of quadratic Lyapunov functions for first-order methods
Pontus Giselsson, Manu Upadhyaya, Sebastian Banert, Adrien Taylor

We present a unifying framework for establishing linear convergence rates for first-order methods, based on quadratic Lyapunov functions. We provide necessary and sufficient conditions for verifying that a quadratic function is a Lyapunov function for the algorithm and problem class under consideration, thereby allowing to produce tight convergence certificates for the setting at hand (i.e., producing worst-case certificates and matching worst-case examples).

■ SD-04

Saturday, 14:30 - 16:10 - Room 1.14

DFO 3 - Global and multiobjective derivative-free optimization

Stream: Derivative-free optimization
Invited session
Chair: *Ana Luisa Custodio*

1 - A Derivative-free Trust-region Approach for Computing Pareto Fronts in Multiobjective Optimization
Aboozar Mohammadi, Ana Luisa Custodio

We propose an algorithm based on a trust-region approach for computing an approximation to the complete Pareto front of a given multiobjective optimization problem. The algorithm alternates between two main steps, the extreme point step, that moves towards extreme points of the Pareto front, and a scalarization step, that attempts to reduce the gaps on the Pareto front, by solving an adequate scalarization problem. In any of these two steps, interpolation models are built to replace the components of the objective function. Numerical experiments illustrate the good performance of the method.

2 - A DIRECT Exploratory-based Hyper-rectangle Evaluation for Bound Constrained Global Optimization Problems
M. Fernanda P. Costa, Ana Maria A.C. Rocha, Edite M.G.P. Fernandes

A DIRECT-type algorithm based on a hyper-rectangle exploratory strategy to identify promising hyper-rectangle is presented. A set of exploratory points are generated on the hyper-rectangles and their average objective function values are used to select promising hyper-rectangles for further partitioned. Thus, the average fitness selection replaces the usual function value at the centre selection if the average fitness is better than that of the centre point. Computational experience on a set of well-known problems shows that the new algorithm is effective and robust.

3 - A Direct Multisearch Filter Method for Biobjective Optimization
Everton Silva, Ana Luisa Custodio

In this work, we propose integrating a filter approach in DMS to address biobjective optimization problems with linear and nonlinear constraints. The violations of the nonlinear constraints are aggregated in a third function and are treated as an additional objective. The algorithm explicitly treats the linear constraints by adapting the positive generating sets considered at each iteration to the geometry of the nearby constraints. We will describe the proposed algorithm, provide results on the theoretical properties of the method, and report numerical experiments.

4 - Incorporating Radial Basis Functions in Global and Local Direct Search
Bruno Baptista, Ana Luisa Custodio, Carmo Bras

GLODS is a global derivative-free optimization algorithm, relying on local directional direct search (DDS) and a clever multistart strategy. New initializations are generated by sampling techniques, not taking advantage of the information gathered by the algorithm in previous iterations. In this work, we propose to use radial basis functions, computed by reusing the function evaluations of DDS, as a strategy for new initializations. We will describe the algorithmic procedure and present numerical results that state the improvement in the performance of GLODS algorithm.

■ SD-05

Saturday, 14:30 - 16:10 - Room 1.15

GO 1 - Advanced techniques for global optimization

Stream: Global optimization
Invited session
Chair: *Sonia Cafieri*

1 - Comparing Different Methods for the Configuration Space Search Problem
Claudia D'Ambrosio, Vanesa Guerrero, Gabriele Iommazzo, Renan Spencer Trindade

In Iommazzo et al (2020), a method composed of two phases was proposed to optimally configure a given algorithm A. In the first phase, machine learning, such as support vector regression, was used to find an approximation of the performance function of A, based on historical data. In the second phase, mixed-integer non-linear programming was used to find the configuration of A that optimizes the approximated performance function. In this work, we focus on the second phase, where we solve the so-called Configuration Space Search Problem. We compare different optimization methods for solving it.

2 - Techniques for accelerating Branch-and-Bound algorithms dedicated to sparse optimization
Gwenaël Samain, Sebastien Bourguignon, Jordan Ninin

We propose two acceleration techniques for sparse optimization (the minimization of a cardinality-penalized least-squares function) with branch-and-bound algorithms. Convex duality is applied to the relaxation problems at each node of the search tree, allowing one to early prune suboptimal nodes thanks to the computation of valid dual bounds. Then, screening methods are implemented during each node evaluation, which reduce the problem size by fixing variables to their optimal value. Numerical experiments study the efficiency of such techniques as a function of the problem difficulty.

3 - On the convergence of controlled mini-batch gradient algorithms
Laura Palagi, Giampaolo Liuzzi, Ruggiero Seccia

We consider the wide-studied problem of minimizing a finite sum of functions. Incremental gradient methods have been deeply studied and usually require strong assumptions for proving convergence. We introduce minimal modifications to control the behaviour of minibatch gradient algorithms. We prove convergence toward stationary points under light and standard assumptions in unconstrained non-convex optimization We show on a set of benchmark problems that the proposed control strategy does not deteriorate the computational performance in comparison with standard incremental gradient methods.

4 - Handling logical constraints by continuous optimization
Sonia Cafieri, Andrew Conn, Marcel Mongeau

We propose a continuous-optimization formulation of logical constraints. Based on the simple idea of guiding the search of a continuous-optimization method towards the parts of the domain where the logical constraint is satisfied, we introduce a smooth penalty-function formulation of logical constraints. The effectiveness of this formulation, that allows the direct use of state-of-the-art continuous optimization solvers, is demonstrated on a real-world application in air transportation.

■ SD-06

Saturday, 14:30 - 16:10 - Room 1.16

VIU 6 - Stochastic approximation

Stream: Variational inequalities and optimization under uncertainty
Invited session
Chair: *Christiane Tammer*

1 - A Variational Inequality Based Stochastic Approximation Approach for Stochastic Inverse Problems.
Akhtar Khan

This talk will focus on the nonlinear inverse problem of estimating stochastic parameters in partial differential equations and variational inequalities with random data. This talk is a joint work with Professor J"urgen Dippon, Joachim Gwinner, and Miguel Sama.

2 - Lp solutions of random differential equations using scales of Banach spaces
Hans-Jörg Starkloff

Extending the theory for scalar-valued differential equations to a corresponding theory for functions with values in Banach spaces some problems arise. For example, investigating Lp solutions of random differential equations often the condition of essential boundedness of a random coefficients is imposed. In the talk it will be illustrated, how such assumptions can be weakened considering equations not in a fixed Banach space but in an appropriate scale of Banach spaces instead. It is a joint work with Antje Mugler from BTU Cottbus.

3 - Randomized stochastic optimization with semi-martingales
Jürgen Dippon

We consider the problem of finding the minimum of a real-valued function whose values can be observed with some noise only. If we assume that these observations are accessible in a time-discrete or -continuous fashion, Kiefer-Wolfowitz-type recursions or Ito-type sde's are appropriate, respectively. These algorithms can be unified in a semi-martingale approach.

Since gradient estimates based on difference quotients may be expensive in high dimensional domains a general randomization scheme with only two observations per time step are introduced, which show optimal rate of convergence.

4 - A Stochastic Nash Equilibrium Problem for Medical Supply Competition
Georgia Fargetta, Laura Rosa Maria Scrimali

We focus on the competition of healthcare institutions for medical supplies described as a two-stage stochastic programming model that includes the unmet demand and the consequent penalty. Institutions simultaneously solve their own stochastic optimization problems and reach a stable state governed by the stochastic Nash equilibrium concept. The problem is formulated as a variational inequality, and both the discrete and the general probability distribution cases are discussed. Some numerical examples applying the progressive hedging method are presented.

■ SD-07

Saturday, 14:30 - 16:10 - Room 1.17

OC 4 - Optimal control and optimization in finance and insurance

Stream: Optimal control and optimization in economics and social sciences
Invited session
Chair: *Fernanda Cipriano*

1 - Mitigating the impact of negative occurrences in investment projects through insurance

Carlos Oliveira, Alexandra Moura

We consider a company that intends to invest in a project, for which are two sources of uncertainty: the company's future revenue and the existence of unexpected adverse events that reduce it. To protect itself against such adverse events, the company can buy an insurance contract. The firm has to decide the moment to invest in the market and the insurance contract that it wants to buy. These decisions depend on the premium required and how the firm measures its risk. In this context, we study the optimal insurance contract, formulating the model as a control problem.

2 - Finite maturity caps and floors on continuous flows under the CEV process

José Carlos Dias, Joao Pedro Nunes, Fernando Silva

This paper offers novel analytical solutions for evaluating perpetual caps and floors on continuous flows under the constant elasticity of variance (CEV) model. We demonstrate that the inclusion of a perpetual bubble value is required to avoid arbitrage opportunities in the case of the CEV process with upward-sloping volatility skews. We then extend the previous literature on caps and floors arrangements by providing new analytical formulae for valuing finite maturity caps and floors that are contingent on continuous flows.

3 - On randomized solutions for optimization problems

Manuel Guerra

Randomized solutions have been introduced to optimization problems, mainly due to the fact that many nonlinear problems become convex when randomized solutions are considered. Many optimization problems in insurance and finance are not convexified by randomized solutions, but spaces of randomized solutions still have useful properties for the study of such problems. Further, randomized solutions can, in many cases, be provided with meaningful economic interpretations and can be used as a tool in model construction.

4 - Portfolio Problem with Consumption Under the α-Hypergeometric Stochastic Volatility Model

Paulo Rocha

In this session, we will look at the dynamic programming method to solve a variant of the portfolio problem in which consumption is allowed. We assume that the agent makes his investment and consumption decisions based on a power utility function. Considering a simple portfolio, composed of a bond and a single stock on a market modeled by the α-Hypergeometric Stochastic volatility model, we will derive the correspondent Hamilton-Jacobi-Bellman equation and discuss the existence of a classical solution and the techniques used to find such a solution.

Saturday, 16:15 - 17:30

■ SE-02

Saturday, 16:15 - 17:30 - Room 1.1

MINO 4 - Decomposition and MINLP for machine learning

Stream: Mixed-integer nonlinear optimization
Invited session
Chair: *Laura Palagi*

1 - Maximum Margin Optimal Classification Trees
Marta Monaci, Laura Palagi, Giorgio Grani
Thanks to their interpretability, decision trees have been intensively studied for classification tasks, and, due to the remarkable advances in mixed-integer programming (MIP), various approaches have been proposed to formulate the Optimal Classification Tree (OCT) problem as a MIP model. We present a novel MIP formulation for the OCT problem, denoted as Maximum Margin Optimal Classification Tree (MARGOT), which exploits the generalization capabilities of support vector machines. In order to improve interpretability, we also develop an embedded feature selection version of our approach.

2 - On multivariate randomized classification trees
Andrea Manno, Edoardo Amaldi, Antonio Consolo
We investigate the nonlinear continuous formulation proposed in Blan- quero et al. (EJOR, vol.284, 2020) for training sparse optimal randomized classification trees, where sparsity is induced by the l1 and l norms. For such trees, we first consider alternative methods to induce sparsity based on concave approximations of the l0 "norm", showing promising results. Then, we derive bounds on the VC dimension. Finally, we propose a simple decomposition training method suited to reduce training times on large- dimensional instances without compromising the accuracy.

3 - Randomized regression trees: a model variant and a decomposition training algorithm
Antonio Consolo, Edoardo Amaldi, Andrea Manno
We present a variant of multivariate randomized regression trees (MRRTs) proposed by Blanquero et al. (EJOR Vol. 299, 2022). The formulation is well-suited not only to decomposition but also to induce fairness measures. Our decomposition training algorithm includes a specific initialization strategy and a heuristic for the reassignment of the input vectors along the branching nodes of the tree. Under mild assumptions, we also establish asymptotic convergence guarantees. Computational results are reported and compared with those of the original version and of an alternative MILO method.

■ SE-03

Saturday, 16:15 - 17:30 - Room 1.2

CGT 6 - Recent advances in projection free methods

Stream: Mathematical programming and computational game theory
Invited session
Chair: *Francesco Rinaldi*

1 - Generalized Self-Concordant Analysis of Frank-Wolfe algorithms
Pavel Dvurechensky, Kamil Safin, Shimrit Shtern, Mathias Staudigl
We propose several variants of the Frank-Wolfe method for minimizing generalized self-concordant (GSC) functions over compact sets. Such problems are ill-conditioned and are motivated by machine learning applications such as inverse covariance estimation or distance-weighted discrimination problems in support vector machines. We obtain O(1/k) convergence rate guarantees in the general situation and linear convergence under strong convexity and additional assumptions.

2 - Projection-free methods for structured problems
Francesco Rinaldi, Immanuel Bomze, Damiano Zeffiro
Recently, projection-free methods have gained popularity thanks to their ability to efficiently handle the structured constraints appearing in machine learning and data science applications. In this work, we focus on some Frank-Wolfe variants for specific classes of structured problems and analyze their theoretical and practical behaviour.

3 - Convergent Mixed-Integer Derivative-Free Optimization
Juan Jose Torres Figueroa, Giacomo Nannicini, Emiliano Traversi, Roberto Wolfler-Calvo
In this project we developed two trust-region algorithms for the optimization of black-box problems with mixed-integer variables. The first algorithm is based on the assumption of local mixed-integer quadratic structure of the objective function, while the second one exploits combinatorial properties in the objective function and works as a hybrid between the DCA and first-order continuous derivative-free methods. Both algorithms are proven to converge into several definitions of mixed-integer first-order stationary points, depending on the structure of the objective function.

■ SE-04

Saturday, 16:15 - 17:30 - Room 1.14

PPA 2 - Primal dual methods and related numerical issues

Stream: Proximal point algorithm and related numerical methods
Invited session
Chair: *Jacek Gondzio*

1 - Non-linear primal-dual descent for Optimal transport and barycenter problems
Antonin Chambolle, Juan Pablo Contreras

I will present a study in collaboration with J.P. Contreras (U. A. Ibanez, Santiago, Chile) where we revisit non-linear primal dual algorithms, and in particular extend an accelerated line-search based primal-dual method of Malitsky and Pock to the non-linear (ie based on a Bregman distance) context. This is done in order to efficiently solve, within some given error, assignment and optimal transport problems, which are large scale constrained linear problems, and leads to state-of-the art results in terms both of efficiency and quality.

2 - Primal-Dual Regularized Interior Point Methods (IPMs): a Proximal Point perspective
Stefano Cipolla, Jacek Gondzio

In this talk, we will show that it is possible to naturally frame the primal-dual regularized IPMs in the context of the Proximal Point Algorithm. Among the benefits of the proposed approach, we will show how convergence can be guaranteed without any supplementary assumptions and how the rate of convergence can be explicitly estimated in relation to regularization parameters. Moreover, we will show how regularization could be exploited in order to devise suitable preconditioners of the Newton system which are required to be re-computed only in a fraction of the total number of IPM

3 - Faster First-Order Primal-Dual Methods for Linear Programming using Restarts and Sharpness
Oliver Hinder

First-order primal-dual methods are often slow at finding high accuracy solutions, which creates a barrier to their use for linear programming (LP). Our work exploits the sharpness of primal-dual formulations of LP instances using restarts in a general setting that applies to ADMM (alternating direction method of multipliers), PDHG (primal-dual hybrid gradient method) and EGM (extragradient method). In particular, we show restarts improve the linear convergence rate of these methods in both theory and practice.

■ SE-05

Saturday, 16:15 - 17:30 - Room 1.15

GO 2 - Simplicial branch and bound

Stream: Global optimization
Invited session
Chair: *Frédéric Messine*

1 - The face graph in simplicial branch and bound
Eligius M.T. Hendrix, Boglárka G.-Tóth, Leocadio G. Casado, Frédéric Messine

Like in Interval Arithmetic, dimension of partition sets can be reduced when monotonicity is detected. One of the challenges is that only border facets are relevant for dimension reduction. With a box constrained feasible set, it is relatively easy to discover a facet is border. However, in a simplicial feasible set, the face graph helps us to identify border facets. We will illustrate this concept up to the extreme of the standard Quadratic Program. This contribution has been supported by The Spanish fund RTI2018-095993-B-I00 in part financed by the European Regional Development Fund (ERDF).

2 - Monotone directions by Linear Programming in simplicial B&B
Boglárka G.-Tóth, Eligius M.T. Hendrix, Leocadio G. Casado, Frédéric Messine

Using interval arithmetic based tests in simplicial branch and bound raise many questions. How to decide the function is monotone over a lower dimensional simplex? If it is monotone, which facets contain the possible minimum? In order to answer these questions with proofs, one can solve some linear programming problems, or a mixed integer programming problem. This does not necessarily pay off, but it is interesting to see how much the best monotone directions help. The research is supported by the Spanish fund RTI2018-095993-B-I00, in part financed by the European Regional Development Fund.

3 - Heuristic directional derivatives in simplicial branch and bound
Leocadio G. Casado, Boglárka G.-Tóth, Frédéric Messine, Eligius M.T. Hendrix

A monotonicity test is not as easy for a simplicial feasible set as for a box, as the orientation does not coincide with the coordinate axes. For a simplicial partition set at the boundary of the feasible region, a question is, which direction (directional derivative) to use in order to discard it? This can be answered by Linear Programming at increasing computational cost. We study applying heuristic direction vectors and their effectiveness. This contribution has been supported by the Spanish fund RTI2018-095993-B-I00 in part financed by the European Regional Development Fund (ERDF).

■ SE-06

Saturday, 16:15 - 17:30 - Room 1.16

COVA 6 - Continuous optimization and variational analysis VI

Stream: Continuous optimization and variational analysis
Invited session
Chair: *Cornel Pintea*

1 - A Single Leader Radner Equilibrium problem: industrial symbiosis in an Eco-Industrial Park
Rossana Riccardi, Elisabetta Allevi, Didier Aussel, Domenico Scopelliti

In this study, we propose a bilevel programming model for an industrial symbiosis network located in an Eco-Industrial Park (EIP). At the upper level, the leader is an authority in charge of the ecological concerns, while, at the lower level, a finite number of enterprises act as followers with economic objectives. Based on the EIP authority decisions, all enterprises compete with each other in a sequential parametric non-cooperative game with the strategies of the EIP authority as exogenous parameters, under uncertainty on resource fluxes and environmental conditions.

2 - Emission trading system and border carbon adjustment: a spatial equilibrium problem for the European pulp and paper industry

Giorgia Oggioni, Elisabetta Allevi, Adriana Gnudi, Igor Konnov, Rossana Riccardi

This paper proposes a spatial equilibrium problem for a pulp and paper supply chain in the context of circular economy. The developed supply chain model accounts for the forest deployment and the production of paper using both virgin pulp and recyclable paper waste, which is collected and re-used. The aim of the paper is to investigate the impacts of carbon emission regulations on the volume produced and exchanged in this supply chain and the related costs. The case study is focused to the European pulp and paper industry that is subject to the Emission Trading Scheme.

3 - Fan-hemicontinuity for the gradient of the norm in Banach space

Marcel Bogdan

Fan-hemicontinuity for the gradient of the norm defined on its scalary-positive subdomain of a Hilbert space was recently proved. For a subdomain expressed as a particular countable intersection of half-planes the gradient of the norm is not Fan-hemicontinuous. The issue is whether or not the positive result on Fan-hemicontinuity can be extended to an arbitrary reflexive Banach space. In this matter it is natural to consider the duality map and its topological properties. Based on weak-weak* sequential continuity of it, the property above holds on lp.

Saturday, 18:00 - 19:00

■ SF-01

Saturday, 18:00 - 19:00 - Auditorium 1C

EUROPT Fellowship Lecture - Oliver Stein

Stream: Plenaries
Invited session
Chair: *Gabriele Eichfelder*
Chair: *Giancarlo Bigi*

1 - Branch-and-bound for continuous and mixed-integer multiobjective optimization

Oliver Stein

The talk explains a recently developed general framework for branch-and-bound methods in multiobjective optimization. It may be applied to continuous and mixed-integer convex and nonconvex multiobjective problems. After providing summaries of main ideas in branch-and-bound and in multiobjective optimization, the talk focuses on natural generalizations of notions and techniques from the single objective to the multiobjective case, including a gap-based termination criterion. As a central tool we discuss convergent enclosures for the set of nondominated points and their limiting behaviour.

Saturday, 19:00 - 19:15

■ SG-01

Saturday, 19:00 - 19:15 - Auditorium 1C

Closing

Stream: Plenaries
Invited session

STREAMS

Complexity of convex optimization

Track(s): 3

Conic optimization

Track(s): 3 4

Continuous optimization and variational analysis

Track(s): 5 6

Derivative-free optimization

Track(s): 4

Global optimization

Track(s): 5

Interval optimization

Track(s): 4

Mathematical programming and computational game theory

Track(s): 3

Mixed-integer nonlinear optimization

Track(s): 2 5

Multiobjective optimization

Track(s): 2

Optimal control and optimization in economics and social sciences

Track(s): 7

Optimization on health informatics

Track(s): 7

Plenaries

Track(s): 1

Proximal point algorithm and related numerical methods

Track(s): 4

Variational inequalities and optimization under uncertainty

Track(s): 6

Session Chair Index

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Friday, 8:30 - 9:00

Friday, 9:00 - 10:00

Friday, 10:25 - 11:40

Friday, 11:45 - 13:00

Friday, 14:30 - 16:10

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