MATERIAL AND GEOMETRICAL PARAMETERS AFFECTING PUNCHING OF REINFORCED CONCRETE FLAT SLABS WITH ORTHOGONAL REINFORCEMENT

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Abstract

The influence of various parameters on the punching behaviour is still in discussion and some contradictory conclusions were drawn by several authors. In order to contribute with new information, a parametric analysis regarding geometrical and material parameters affecting punching behaviour of flat slabs with orthogonal reinforcement and square columns was run in a previously verified 3D nonlinear finite element analysis. It was found that compressive strength, fracture energy and reinforcement ratio are the most important parameters regarding punching load. Nevertheless, although tensile strength and modulus of elasticity don't affect punching load, they are important regarding punching behaviour in terms of cracking and stiffness. It was shown that the concept of structural brittleness is appropriate to be used in punching load provisions by codes, turning possible its use in a broad range of concrete types.

Keywords: Punching, orthogonal reinforcement, 3D nonlinear finite element analysis, parametric analysis

1   Introduction

The influence of several parameters in punching behaviour is still in discussion, since from the few parametric studies using nonlinear analysis that have been performed, contrary results have outcome. Parametric studies became possible with the development of computational capacities

and the developing of knowledge of nonlinear behaviour of concrete, both in compression and tensile, in the last thirty years. From the studies previously developed regarding parametric analysis of material factors affecting punching behaviour in RC flat slabs, Menétrey [1], Hallgren [2] and Ozbolt *et al.* [3] works stand out. The first two were performed using two-dimensionally modelled systems using rotationally symmetric elements, while the last one was done using a three-dimensional finite element code, based in the microplane material model. Although two-dimensionally modelled systems using rotationally symmetric elements are suitable for slabs with ring reinforcement, Menétrey [1] and Hallgren [2], used it to analyse orthogonal reinforced concrete slabs, adopting special conditions. The curves of the finite element analysis were stiffer than experimental ones, and both justified it from the simplified modelling of the two-way reinforcement as an axi-symmetric mesh, thus turning its results questionable. Nevertheless, at the time, 3D analysis was difficult to perform because computer capabilities were relatively weak. Latter, Ozbolt *et al.* [3] performed a 3D finite element analysis of a punching test on an interior slab-column connection, based in the microplane model for concrete and a smeared crack approach, concluding that the code is able to realistically predict the load-bearing behaviour. Nevertheless, from the presented results only peak load and corresponding displacement are well estimated. The same also happened in terms of comparison of steel strains.

Regarding parametric analysis performed in the mentioned previous works, the main conclusions were: Ménetrey [1] concluded that punching failure is due to tensile failure of concrete along the inclined punching crack and is not due to compressive failure of concrete and that increasing the reinforcement ratio implies increasing the punching load. Size effect was also investigated, resulting in a nominal shear stress decrease with increasing thickness of the slab; Hallgren [2] showed that tensile strength has a major influence on punching capacity and compressive strength of concrete has a small influence. Showed also that the greater brittleness of higher concrete grades reduces the increase of the punching strength, and found a linear correlation between numerically obtained ultimate loads and the tensile strength multiplied with the ratio between characteristic length of concrete (l_{ch}), proposed by Hillerborg *et al.* [4] and the depth of compression zone; Ozbolt *et al.* [3] concluded that fracture energy and reinforcement ratio have a similar and relatively strong influence on the maximum load, stating that high values of fracture energy (G_F) and reinforcement ratio lead to a more stable growth of the cracks in the tensile cord, and as a result, the height of the compression zone increases and leads to higher punching load and more brittle failure. The numerical results also indicate a relatively small effect of the compressive and tensile concrete strength on the punching capacity. They stated that depending on the fracture energy, the crack propagation at peak load might even become less stable if tensile strength increases, and therefore the positive effect of tensile strength remains relatively small while brittleness of the structural response tend to increase with higher tensile strengths. It was also demonstrated that increasing effective depth leads to a decrease of the nominal shear capacity of the slabs. Regarding Ozbolt *et al.* [3] results, Menétrey [5] stated that their conclusion, that tensile and compressive strength doesn't influence punching failure is not justified experimentally and this may result from the assumption of an uniaxial stress strain response according to the microplane model for witch the biaxial and triaxial concrete response are neglected.

Bearing in mind the contradictions between the results obtained by the above mentioned investigators, and since they assumed certain simplifications, namely, using two-dimensionally modelled systems using rotationally symmetric elements for orthogonal reinforcement and the limitations of the microplane model, there is a need for a parametric analysis based in 3D analysis using more recent and developed material behaviour models turning possible to draw reliable conclusions. The model to be used in the parametric analysis was previously verified by comparison with several reliable experimental tests presented in Faria *et al.* [6], showing good agreement in terms of failure loads and reasonable agreement in terms of pre-punching failure behaviour. The results of the parametric analysis are important to further understand the punching phenomenon and turn possible the improvement of existing provisions, namely, regarding the introduction of fracture mechanics parameters, like the characteristic length of concrete (l_{ch}), reflecting influence of brittleness of concrete, based on fracture energy, elasticity modulus (E_c) and

tensile strength (f_{ctm}) of concrete, as first proposed previously by Hillerborg *et al.* [4]. Fracture mechanics parameters have already been introduced in Hallgren's [2] proposed mechanical model and Staller's [7] empirical model, for the determination of punching capacity.

2 Experimental Investigation

2.1 Tested models

Reference slabs used in this work were named DF1 and DF4 and they had dimensions of 2300x2300 mm^2 with 100 mm thick and 120 mm thick, respectively. The punching load was applied by a hydraulic jack positioned under the slab, through a steel plate with 200x200 mm^2 in the centre of the slabs and eight points on the top of the slab were connected to the strong floor of the laboratory using prestress strands and spreader beams. To assess the strength of the concrete used in the production of the test specimens, compression tests on cubes measuring 150x150x150 mm^3 were carried out (f_{ccm}) in the test day. Values for other concrete properties were obtained with the relations presented in MC90 [8]. The introduction of fracture mechanics in the improvement of punching provisions is one of the objectives of this work. This intends the improvement of punching load prediction since it turns possible to take into account size and type of aggregate and also the presence of fibres in concrete.

Within nonlinear fracture mechanics, a characteristic length of the material l_{ch} (Eq. (1)) may be defined through a relation between the prefracture properties of the material and its postfracture tensile softening properties (Gustafsson *et al.* [10]). The characteristic length is not a length in any physical sense, but it gives the relation between the fracture and elastic characteristics of the material. So, it is a measure of the brittleness of the material, and for a specimen of the same size and shape, a shorter l_{ch} indicates that the material is more brittle. Fracture energy G_F, defined as the energy required to propagate a tensile crack of unit area, according to Hilsdorf *et al.* [11], may be estimated by equation (2), from MC90 [8], equivalent to the one obtained according to RILEM recommendation [12] The equation presented depends on the maximum size of aggregate and in the strength grade of concrete expressed by its compressive strength. As this equation was directed towards designers, the relation has been formulated such that only parameters generally known to the designer are used, and so type of aggregate and other admixtures such as fibre reinforcement weren't taken into account. According to Hilsdorf *et al.* [11], equation (2) is best documented for maximum aggregate sizes (d_a) between 12 mm and 20 mm, with a correlation coefficient of 0.83, suggesting that for remaining maximum aggregate sizes, G_F should be determined experimentally, for more accurate predictions.

$$l_{ch} = \frac{E_c \cdot G_F}{f_{ctm}^2} \tag{1}$$

$$G_F = G_{F0}\left(\frac{f_{cm}}{10}\right)^{0.7} \text{ with } G_{F0} = \begin{cases} 25 \\ 30 \\ 38 \end{cases} \text{ for } d_a = \begin{cases} 8\,\text{mm} \\ 16\,\text{mm} \\ 32\,\text{mm} \end{cases} \tag{2}$$

3 Finite Element Modelling

3.1 Material modelling

In the finite element analysis (FEA) it was used the ATENA [13] program. It uses realistic constitutive model of concrete behaviour that enables a simulation of the real structure behaviour in service as well as in ultimate loading conditions. The numerical model uses three-dimensional isoparametric elements with eight nodes and only one quarter of the slab was modelled using symmetry, for simplification. The redistribution of internal forces due to the nonlinear material

behaviour is taken into account and the resulting stress and deformation state satisfies all three requirements of mechanics: equilibrium of forces, compatibility of deformations and material laws.

The material chosen to simulate concrete was *CC3DNonLinCementitious2*, that consists in a combined fracture-plastic model for concrete. Tension is handled by a fracture model, based on the classical orthotropic smeared crack formulation and the crack band approach. It employs the Rankine failure criterion, exponential softening, and it can be used as a rotated or a fixed crack model. In this study it was adopted the fixed crack model. The plasticity model for concrete in compression is based on the Menétrey–Willam [14] failure surface. The position of the Menétrey-Willam [14] failure surface is not fixed but it can expand and move along the hydrostatic axis, simulating the hardening and softening stages. The model can be used to simulate concrete cracking, crushing under high confinement and crack closure due to compression in other directions. The fictitious crack model developed by Hillerborg *et al.* [4] is considered, in witch the tensile strength is controlled by the crack opening. The softening behaviour of concrete in tension adopted is the one derived experimentally by Hordjik [15], and it is used an enhanced crack model that was originally proposed by Bazant *et al.* [16]. Aggregate interlock and dowel action of the reinforcement is taken in account by a reduction of the shear modulus with growing strain normal to the crack, according to the law derived by Kolmar [17]. The reinforcement can be included in two ways, either as smeared, or as discrete bar elements. In this study, discrete bar elements were used, and it is represented by uniaxial ideally elasto-plastic stress-strain relationship. Bond is also taken into account, and in this study it is adopted the MC90 [8] bond/slip law for the reinforcement bars. In this nonlinear FEA, the load was increased by deformation control in steps according to the standard Newton-Raphson iteration method.

3.2 Finite Element Mesh

A quarter of the slab was modelled using symmetry, for simplification. The punching failure zone was discretized with a fine mesh of brick finite elements, being the outer zone meshed using shell elements. The load on the slab was introduced by deformation control thru a steel plate in deformation steps of 0.2 mm. **Fig. 1** represents one of the models used in this study.
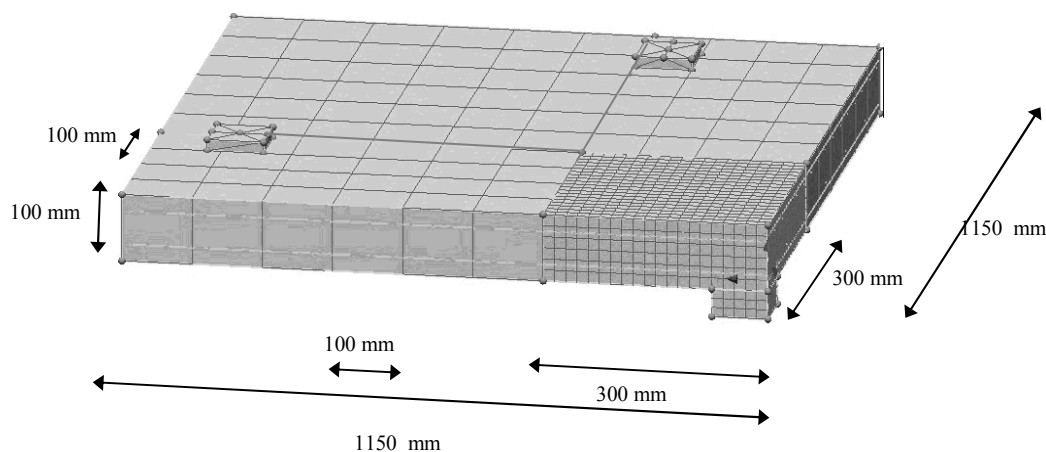

Fig. 1 Three dimensional mesh discretization (model DF1)

In the case of 100 mm thick slab (DF1) the discretization of the punching zone was done with 20x20x20 mm^3 bricks within an extension of 300 mm, and in the outer zone were used 100 mm thick shells divided in six layers, with the larger dimension never exceeding 100 mm, in order to improve calculation times, and since deformation is concentrated near the column, there is no need to use better accuracy elements outside the punching area. In the 120 mm thick slab (DF4) the discretization of the punching zone was done with 20x20x20 mm^3 bricks within an extension of 400 mm, and in the outer zone were used 120 mm thick shells, divided in six layers, with the larger dimension never exceeding 125 mm.

4

4   Parametric Analysis

4.1   Influence of concrete properties

Although concrete parameters such as tensile strength, compressive strength and fracture energy are known to be correlated, in the following, they are considered independent (when one changes, all the others remain constant), in order to determine their individual influence and effect in the punching phenomenon. For that reason the results from this analysis may not be totally true, but they give a strong indication of what are the main effects regarding punching failure behaviour. So, it was studied the effect of tensile strength, compressive strength and fracture energy variation, and the results compared with the results from the reference models described. Each of these parameters was considered as 50%, 70%, 140% and 200% of the reference models.

4.1.1 Tensile Strength and Fracture Energy

In this numerical analysis (**Fig. 2 (a))**, is possible to observe that tensile strength has no significant influence on the punching load. There was even a decrease in average of about 10% on the punching load with a 100% increase of the tensile strength, relating to reference model tensile strength. As all other variables were maintained constant, this is related to material brittleness, as with an increase in tensile strength, stored elastic energy grows in relation to fracture energy, and the response becomes more brittle, and punching load decreases with decreased ultimate displacements. Also, it has an influence in cracking loads in the earlier phase of loading. In **Fig. 2 (b)** is also possible to understand the effect of fracture energy variation on the punching load, showing the tendency line and respective correlation factor. It is shown that the punching resistance increases approximately as the cube root function of fracture energy.
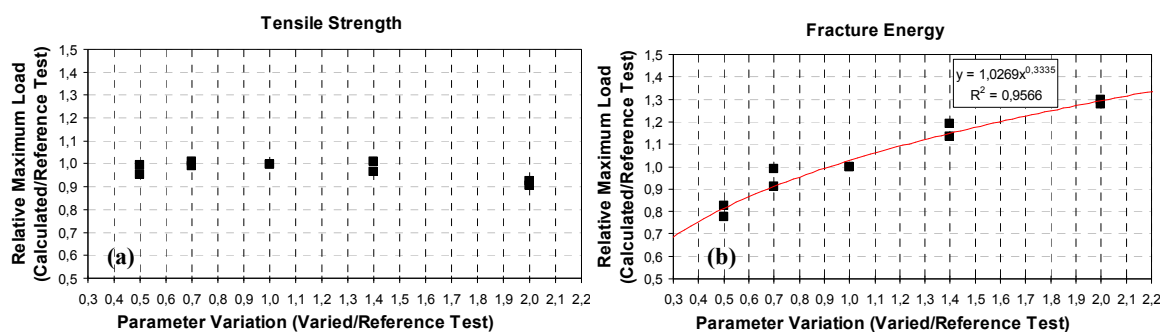

Fig. 2   Influence of (a) tensile strength and (b) fracture energy variation on punching load

Once more, regarding material brittleness defined before, it is seen that with increased fracture energy, the material ductility increases, corresponding to the obtained results. Comparing to other investigations, the results presented so far match approximately with those from Ozbolt *et al.* [3], as the results from Menétery [1] and Hallgren [2] showed a strong influence of tensile strength. This may result from the fact that, both numerical analysis, applied a 2D axi-symmetric analysis to simulate orthogonal reinforced concrete slabs, taking in account specific conditions.

4.1.2 Modulus of Elasticity and Compressive Strength

As in the tensile strength variation, increasing the modulus of elasticity had a slight influence on the punching load capacity (**Fig. 3 (a)**). More relevant is that the punching capacity increases approximately as the cube root function of compressive strength, as shown in **Fig. 3 (b)**. These results match EC2 [9] provisions, once in that code the punching load is also function of the cube root of concrete compressive strength.
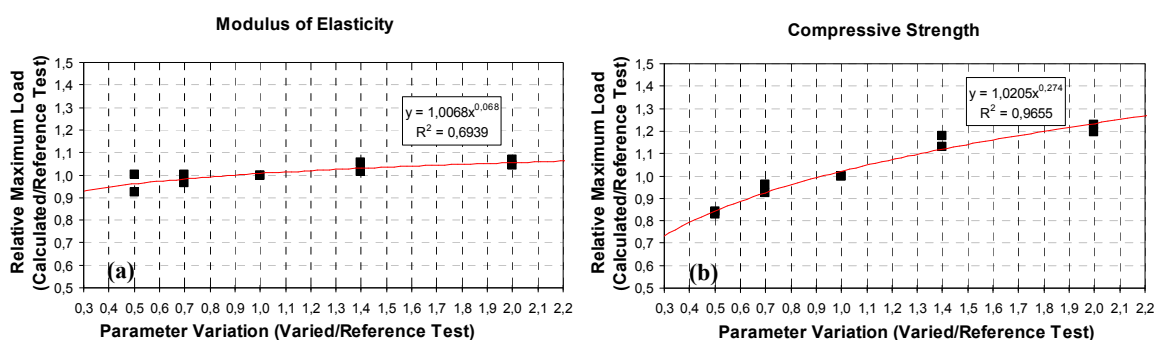
Modulus of Elasticity

Compressive Strength

Fig. 3 Influence of the (a) modulus of elasticity and (b) compressive strength on punching load

Regarding other works mentioned before, this is the first study where compressive strength is shown to have a significant influence in punching loads. Nevertheless, these results aren't unexpected, since in the first mechanical model to yield good agreement with test results, from Kinnunen/Nylander [18], the failure is mainly related to a highly stressed compression zone and the load is transferred to the column by an inclined strut. Their failure criterion was expressed in terms of a limitation of the inclined radial compressive stress and a limitation of the tangential compressive strain at the shear crack. Their work was followed by other researchers, that improved their model, and introduced fracture mechanics parameters, improving its agreement with experimental data, such as Hallgren's [2] work, where failure criteria is connected with tangential compressive strains in the bottom of the slab and vertical tensile strains across its depth, that are in turn related with fracture mechanics concepts.

4.2 Reinforcement Ratio

The reinforcement ratio has a significant influence on the punching loads (**Fig. 4**) since it is shown that the punching resistance increases approximately as the cube root function of reinforcement ratio. This results are in agreement with the provisions of EC2 [8] once in its expression punching load is also function of the cube root of reinforcement ratio. According to Ozbolt *et al.* [3] that registered the same results relating to reinforcement ratio influence, this increase leads to a higher height of the compression zone leading to higher punching loads accompanied by more brittle failures. As in the fracture energy and compressive strength cases, it is shown that the punching resistance increases approximately as the cube root function of reinforcement ratio, as shown in **Fig. 4**.
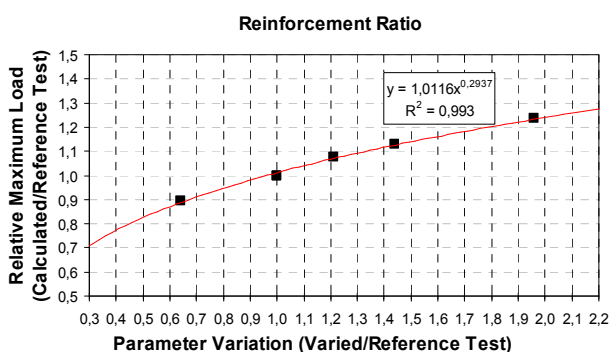
Reinforcement Ratio

Fig. 4 Influence of reinforcement ratio variation on punching load.

5 Influence of material/structural brittleness

5.1 EC2 provisions

From the results obtained in section 4, it was observed that EC2 [9] (Eq. (3) to (6)) provisions agrees with the obtained results, except for the size effect parameter $1+(200/d)^{1/2}$, since it only

depends on average effective depth d (based in effective depth of both orthogonal directions d_y and d_z). In EC2 [9] the punching resistance is essentially function of the column geometry, the slab thickness, concrete compressive strength, amount of steel reinforcement (average-ρ, based in reinforcement ratio of both orthogonal directions ρ_{ly} and ρ_{lz}), the existence of shear reinforcement and the existence or not of prestress. EC2 [9] provisions regarding punching load are the same as for MC90 [8], except that it limits k to a maximum of 2.0 and ρ to 2%. Nevertheless, in this work, these limits aren't taken in account. For the calculation of the unfactored resistance without punching shear reinforcement, using EC2 [9], neglecting compression effects, the following expression was used:

$$V_{Rm} = \left(0.18 \cdot k \cdot \left(100 \cdot \rho \cdot f_{cm}\right)^{1/3}\right) \cdot u \cdot d \tag{3}$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2 \quad \text{(d in mm)} \tag{4}$$

$$\rho = \sqrt{\rho_{ly} \cdot \rho_{lz}} \leq 0.02 \tag{5}$$

$$d = \frac{d_y + d_z}{2} \tag{6}$$

In the quantification of the punching resistance the mean values of the materials strengths, without partial coefficients, were used. V_{Rm} is the predicted failure load.

As mentioned before, several investigators showed a relation between shear strength and material/fracture brittleness (Gustafsson *et al.* [10], Hallgren [2], Staller [7]). The brittleness of the material may be measured using a characteristic length of the material l_{ch} (Eq. (2)), while the brittleness of the structure may be defined as the ratio between structural size, usually defined the effective depth d, and l_{ch}, according to Eq. (7). It is the ratio between the stored elastic energy in the structure and the dissipated energy during fracture process (Noghabai [19]). In other words, the size effect is due to release of strain energy from the structure into the fracture zone as the cracking extends. The larger the structures, the greater is the energy release and the more fracture energy is needed to avoid a sudden failure. When eventually, the tensile strength is exceeded and a critical diagonal crack form, the residual stresses acting across the crack have to resist the impulse load that is connected to the energy release. Hence, the requirements on the tensile properties of concrete will increase as the size increases.

$$\frac{d}{l_{ch}} = \frac{d \cdot f_{ctm}{}^2}{E_c \cdot G_F} \tag{7}$$

A higher value of d/l_{ch} corresponds to a more brittle structure, in structures sensitive to tensile stress-induced fracture. Gustafsson *et al.* [10] suggested that in expressions regarding shear strength, where the size effect depends on effective depth d, this d should be replaced by d/l_{ch}.

Nevertheless, based on the results obtained in the parametric analysis regarding tensile properties of concrete, it is clear that only fracture energy has an important role in punching load, contrary to tensile strength and modulus of elasticity, that by themselves practically have no influence. So, in this case, the punching capacity should be proporcional to

$$\frac{d}{l_{ch}} \cdot \frac{E_c}{f_{ctm}{}^2} = \frac{d}{G_F} \tag{8}$$

meaning that elastic properties of concrete in tension aren't important regarding punching load.

Also, from parametric analysis regarding compressive strength and reinforcement ratio, it was concluded that both of them shown that the punching resistance of slabs increases approximately with their cubic root. This matches with the EC2 [9] expression (Eq. (3)). So, it

appears that MC90 [8] expression gives the best approximation with the obtained results, relating to those parameters. But EC2 [9] expression doesn't take in account, at least, in a accurate way, the influence of material/structure brittleness, since in its size effect parameter the only variable is the effective depth (d). Based on the results presented in this work, and in the evaluation of several experimental results, it is proposed a substitute to the size effect parameter of EC2 [9] expression, based on fracture mechanics parameters allready discussed before. This proposal is useful, namely in the use of concretes composed of different types and sizes of aggregates and in fibre incorporated concrete, different from those where the EC2 [9] expression was calibrated, in order that the proposed change covers and may be used in a wider range of different concrete composition. In the change proposal the size effect parameter $1+(200/d)^{1/2}$ will be replaced, making possible to attain more accurate predictions and less scatter in the results.

5.2 Fracture mechanics parameters/size effect replacement

The first step to develop the changed expression was the numerical analysis of several models with different material properties, with different effective depths and different reinforcement ratios, based in models DF1 and DF4. This analysis is necessary to determine an accurate relation between FEA punching load (V_{FEA}) and G_F/d. Based on the results it is possible to plot values of normalized shear stress (Eq. (9)), against G_F/d (**Fig. 5**).

$$\frac{V_{FEA}}{0.18\cdot\left(100\cdot\rho\cdot f_{cm}\right)^{1/3}\cdot u\cdot d} \tag{9}$$

From the models run with different effective depths, ranging from 69 mm to 208 mm, the results showed a continuously increase in shear stress as the effective depth decreased until a minimum of 69 mm. Also, from parameters analysis related with reinforcement ratio, it was observed an increase in punching load even for reinforcement ratios greater than 2%.
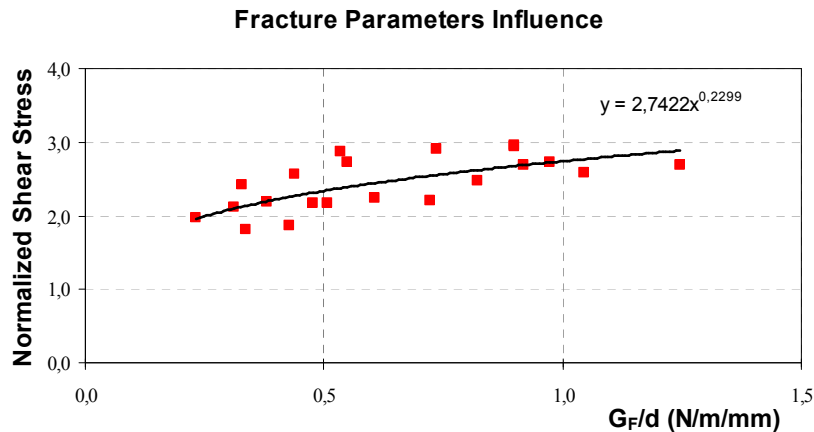

Fracture Parameters Influence

$y = 2,7422x^{0,2299}$

Fig. 5   Relation between normalized shear stress and G_F/d

The relation between normalized shear stress and G_F/d is shown in **Fig. 5**, based in a regression analysis, and for reasons of simplification is assumed as $2.72(G_F/d)^{0.25}$. This value includes interaction between compression strength and fracture energy and includes the effect of different slenderness, varying between 4 and 13, and thus covering the most usual slab slenderness. The proposed equation is:

$$V_{Rm} = 0.49\cdot\left(\frac{G_F}{d}\right)^{0.25}\cdot\left(100\cdot\rho\cdot f_{cm}\right)^{1/3}\cdot u\cdot d \tag{10}$$

Afterwards, the changed expression, taking in account the results obtained in [6], was compared with a set of more than 40 tests results from different experimental investigations including slabs

with high strength and lightweight concrete, showing a good approximation. The average relation between the experimental and computed (Eq. (10)) punching loads was of 0.96 with a COV (coefficient of variation) of 0.11. Staller's [7] and EC2 [9] expressions gave average relations of 0.93 and 1.14, with a COV of 0.12 and 0.10, respectively.

6   Conclusions

Once there isn't still a consensus on the role of concrete parameters on the punching capacity of flat slabs, this work intends to contribute to its knowledge. Based on a finite element model capable of simulating 3D nonlinear behavior of reinforced concrete elements previously verified [6], it was developed a parametric analysis in order to determine the main parameters that influence the punching load in reinforced concrete flat slabs, regarding concrete properties and reinforcement ratio, and the main conclusions were:

- Tensile strength and modulus of elasticity have small influence on the punching load;
- Compressive strength and fracture energy are the main material properties that influence punching load, since both of them showed that punching load increase approximately with their cube root;
- Reinforcement ratio has also an important role, as punching load also increases approximately with its cube root.

Based on the results, EC2 [9] expression agrees with the obtained results, except for the size effect parameter $1+(200/d)^{1/2}$, once it only depends on effective depth d. So, it is suggested a substitute for the size effect parameter k, where fracture mechanics parameters are introduced, namely, structure brittleness (d/l_{ch}). This parameter allows taking directly in account the type and size of aggregates or the presence of fibres in concrete.

Although not presented here, the tensile strength and the modulus of elasticity of concrete, have an influence in the behaviour during loading of the slabs, as the tensile strength influences cracking loads and the modulus of elasticity influences the response stiffness.

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